

Theory and Phenomenology of Type I strings and M-theory[†]

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Abstract

The physical motivations and the basic construction rules for Type I strings and M-theory compactifications are reviewed in light of the recent developments. The first part contains the basic theoretical ingredients needed for building four-dimensional supersymmetric models, models with broken supersymmetry and for computing low-energy actions and quantum corrections to them. The second part contains some phenomenological applications to brane world scenarios with low values of the string scale and large extra dimensions.

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1. Introduction

Since the discovery of the anomaly cancellation for superstrings in ten dimensions (10d) [1], the construction of the heterotic strings [2] and the seminal papers on the compactification to four dimensions [3], [4], string theory has become the best candidate for a fundamental quantum theory of all interactions including Einstein gravity. The theory contains only one free dimensionful parameter, the string scale M_s , while the four-dimensional (4d) gauge group and the matter content are manifestations of the geometric properties of the internal space that, however, we are unable to select in a unique fashion. The Standard Model hopefully would correspond to a particular internal space or vacuum configuration chosen by nature by some still unknown mechanism. The 4d low-energy couplings depend only on the string scale and on various vacuum expectation values (vev's) of fields describing the string coupling constant, the size and the shape of the internal manifold. There is therefore, in principle, the hope to understand the empirically observed pattern of the parameters in the Standard Model. A long activity in heterotic strings [5], [6] was devoted to this program [7], in the hope that these rather tight constraints would determine in some way the right vacuum describing our world. Despite serious insights into the structure and the phenomenological properties of 4d models [5, 6], no unique candidate having as low-energy limit the Standard Model emerged. Moreover, there were (and there still are) conceptual problems to be solved, as for example the large degeneracy of the string vacua and the related problems of spacetime supersymmetry breaking and dilaton stabilization. Most of these problems asked for a better understanding of the strong coupling limit of string theory, of which very little was known for a long time. The other string theories were, for a long time, discarded as inappropriate for phenomenological purposes. Type II strings were considered unable to produce a realistic gauge group, while Type I strings, despite serious advances made over the years [8, 9, 10, 11], that revealed striking differences with respect to heterotic strings, were less studied and their consistency rules not widely known

as for heterotic vacua.

By the middle of the last decade, it became clear that all known string theories are actually related by various dualities to each other and to a mysterious eleven dimensional theory, provisionally called M-theory [12]. It therefore became possible to obtain some nonperturbative string results, at least for theories with enough supersymmetry. Moreover, the discovery and the study of D-branes [13] put the duality predictions on a firmer quantitative basis and, on the other hand, was an important step in unravelling the geometric structure underlying the consistency conditions of Type I vacua, stimulating a new activity in this field [14]. The first chiral 4d Type I model was proposed [15], and efforts on 4d model building allowed a better understanding of supersymmetric 4d vacua [16] and of their gauge and gravitational anomaly cancellation mechanisms [17], similar to the 6d generalized Green-Schwarz mechanism discovered by Sagnotti [18]. The presence of D-branes in Type I models led to new mechanisms for breaking supersymmetry by compactification [19, 20], by internal magnetic fields [21, 22] or directly on some (anti)branes [23, 24, 25], providing perturbatively stable non-BPS analogs of Type IIB configurations [26].

On the phenomenological side, the M-theory compactification of Horava and Witten [27], with a fundamental scale $M_{11} \sim 2 \times 10^{16}$ GeV, provided a framework [28] for the perturbative MSSM unification of gauge couplings [29], and stimulated studies of gaugino condensation [30], of 4d compactifications [31, 32, 33] and of supersymmetry breaking along the new (eleventh) dimension [34, 32]. Moreover, it was noticed [35] that in Type I strings the string scale can be lowered all the way down to the TeV range. Similar ideas appeared for lowering the fundamental Planck scale in theories with (sub)millimeter gravitational dimensions [36], as an alternative solution to the gauge hierarchy problem, and, simultaneously, a new way for lowering the GUT scale in theories with large (TeV) dimensions [37] was proposed. The new emerging picture found a simple realization in a perturbative Type I setting [38] with low string scale (in the TeV range) and became

the subject of an intense activity, mostly on the phenomenological side, but also on the theoretical side.

The goal of the present paper is to review the ideas which led to this new picture and to present a comprehensive introduction to the basic string tools necessary for understanding the corresponding physics. The convention for the metric signature throughout the paper is $(-, +, \dots +)$, ten-dimensional (10d) indices are denoted by A, B, \dots , eleven dimensional indices by I, J, \dots , five-dimensional (5d) indices by M, N, \dots and four-dimensional indices (4d) by μ, ν, \dots .

2. From heterotic strings to Type I strings and M-theory

To date, the (super)strings are the only known consistent quantum theories including Einstein gravity. They are therefore promising candidates for a unifying picture of elementary particles and fundamental interactions.

It has been known for a long time that there are five consistent (anomaly-free) superstring theories in 10d, namely:

- The heterotic closed string theories, with gauge groups $SO(32)$ and $E_8 \times E_8$ and $\mathcal{N} = 1$ spacetime supersymmetry, that after a toroidal compactification corresponds to $\mathcal{N} = 4$ supersymmetry in four dimensions. There are also nonsupersymmetric heterotic vacua, in particular a non-tachyonic one based on the gauge group $SO(16) \times SO(16)$ [39, 40].

- The (non-chiral) Type IIA and (chiral) Type IIB closed string theories, with $\mathcal{N} = 2$ spacetime supersymmetry, that after a toroidal compactification corresponds to $\mathcal{N} = 8$ supersymmetry in four dimensions. Different modular invariant GSO projections in 10d give rise to nonsupersymmetric theories, called 0A and 0B [41].

- The Type I open string theory, with gauge group $SO(32)$ and $\mathcal{N} = 1$ supersymmetry. In this case the (Chan-Paton) gauge quantum numbers sit at the ends of the string and

allow, in more general cases, to construct the gauge groups $O(n)$, $USp(n)$ and $U(n)$ [42]. This theory can be defined as a projection (orientifold) of the Type IIB string [8]. Analogously, (nonsupersymmetric) orientifolds of Type 0A and 0B can be constructed [10], in particular a nontachyonic 0B orientifold with gauge group $U(32)$ [43].

The massless modes of the above superstring theories and their interactions are described by effective 10d supergravity theories, namely:

- The low energy limit of the the two heterotic theories *and* of the Type I open string are described by the ten dimensional $\mathcal{N} = 1$ (or $(1,0)$) supergravity coupled to the super Yang-Mills system based on the gauge groups $SO(32)$ and $E_8 \times E_8$, respectively.

- The low energy limit of the Type II strings is described by the $\mathcal{N} = 2$ Type IIA supergravity (with $(1,1)$ supersymmetry) and Type IIB supergravity (with $(2,0)$ supersymmetry).

The common features of all the effective ten dimensional superstring theories is the presence of supersymmetric multiplets containing in the bosonic sector the graviton $g_{\mu\nu}$, the dilaton ϕ and an antisymmetric tensor $B_{\mu\nu}$. The string coupling constant is a dynamical variable $\lambda = \exp(\phi)$, and the only free parameter is the string length $\alpha' = 1/M_s^2$, where M_s is the string mass scale.

The 4d theories are defined after a compactification similar to the old Kaluza-Klein scenario. Typically, the ten dimensional spacetime is decomposed as $M_{10} = M_4 \times K_6$, where M_4 is our four dimensional Minkowski spacetime and K_6 is a compact manifold whose volume V traditionally defines the compactification scale M_c

$$V = M_c^{-6} \equiv M_{GUT}^{-6} , \tag{1}$$

the scale of the Kaluza-Klein mass excitations in the internal space. The compactification scale was also identified above with the grand unified scale M_{GUT} in the string unification picture, because the field theory description breaks down above M_c . However, we will see

later that (1) can be substantially altered in some string models.

The massless fields in a toroidal compactification are the zero modes of the 10d fields, that in more general settings depend on the topology of the compact space K_6 . If we denote by i, j six dimensional internal indices, then we have, for example, the following decompositions:

$$g_{AB} : g_{\mu\nu} \ g_{ij} \ g_{\mu i} \ ,$$

$$B_{AC} : B_{\mu\nu} \ B_{ij} \ B_{\mu i} \ ,$$

where in 4d $g_{\mu\nu}$ is the graviton, $g_{\mu i}$, $B_{\mu i}$ are gauge fields and g_{ij} are scalars describing the shape of the compact space. On the other hand, $B_{\mu\nu}$ and B_{ij} are pseudoscalar, axion-type fields.

Four dimensional string couplings and scales are predicted in terms of the string mass scale M_s and of various dynamical fields: dilaton, volume of compact space, etc. In contrast to the usual GUT models, which do not incorporate gravity and thus make no predictions for Newton's constant, the perturbative string models do make a definite prediction for the gravitational coupling strength. Since the length scale of string theory $\sqrt{\alpha'}$, the volume V of the internal manifold and the expectation value of the dilaton field ϕ are not directly known from experiment, one might naively think that by adjusting α' , V , and $\langle\phi\rangle$ one could fit to any desired values the Newton's constant, the GUT scale M_{GUT} , and the GUT coupling constant α_{GUT} . However, this is not true for the weakly coupled heterotic strings. In 10d, the low energy supergravity effective action looks like

$$S_{eff} = \int d^{10}x \sqrt{g} \ e^{-2\phi} \left(\frac{4}{(\alpha')^4} R - \frac{1}{(\alpha')^3} \text{tr} F^2 + \dots \right) \ , \quad (2)$$

where R is the scalar curvature and $\text{tr} F^2$ is the Yang-Mills kinetic term. After compactification on an internal manifold of volume V (in the string metric), one gets a four-dimensional effective action that looks like

$$S_{eff} = \int d^4x \sqrt{g} \ e^{-2\phi} V \left(\frac{4}{(\alpha')^4} R - \frac{1}{(\alpha')^3} \text{tr} F^2 + \dots \right) \ . \quad (3)$$

Notice that the same function $Ve^{-2\phi}$ multiplies both R and $\text{tr}F^2$. From (3), defining the heterotic scale $M_H = \alpha'^{-1/2}$, one thus gets

$$M_H = \left(\frac{\alpha_{GUT}}{8}\right)^{1/2} M_P, \quad \lambda_H = 2(\alpha_{GUT}V)^{1/2} M_H^3, \quad (4)$$

where $M_P = G_N^{-1/2}$ is the Planck mass. Then $M_H \sim 5 \times 10^{17}$ GeV, and therefore there is some (slight) discrepancy between the GUT scale M_{GUT} and the string scale M_H . Indeed, from (1) and (4) we find $M_{GUT}/M_H = (4\alpha_{GUT}/\lambda_H^2)^{1/6}$ which asks, in order to find $M_{GUT} \sim 2 - 3 \times 10^{16}$ GeV, for a very large string coupling λ_H . The problem might be alleviated by considering an anisotropic Calabi-Yau space and a lot of effort in this direction was made over the years [7].

The above picture evolved considerably in the last few years. First of all, it was a puzzle that the heterotic $SO(32)$ and the Type I ten dimensional strings share the same low-energy theory. Indeed, the two low-energy actions coincide if the following identifications are made

$$\lambda_I = \frac{1}{\lambda_H}, \quad M_I = \frac{M_H}{\sqrt{\lambda_H}}, \quad (5)$$

where M_I, M_H are the heterotic and Type I string scales and λ_I, λ_H are the corresponding string couplings. A natural conjecture was made, that the two string theories are dual (in the weak-coupling strong-coupling sense) to each other [45, 46]. New arguments in favor of this duality came soon:

- The heterotic $SO(32)$ string can be obtained as a soliton solution of the Type I string [47].

- There is a precise mapping of BPS states (and their masses) between the two theories. If we compactify, for example, both theories to nine dimensions on a circle of radius $R_I(R_H)$ in Type I (heterotic) units, we can relate states with the masses

$$\mathcal{M}_I^2 = l^2 R_I^2 M_I^4 + \frac{m^2 R_I^2 M_I^4}{\lambda_I^2} + \frac{n^2}{R_I^2} \leftrightarrow \mathcal{M}_H^2 = m^2 R_H^2 M_H^4 + \frac{l^2 R_H^2 M_H^4}{\lambda_H^2} + \frac{n^2}{R_H^2}, \quad (6)$$

where n, l (n, m) are Kaluza-Klein and winding numbers on Type I (heterotic) side. It is interesting to notice in this formula how perturbative heterotic winding states (m) become

non perturbative on the Type I side. An important role in checking dualities in various dimensions is played by extended objects called Dirichlet (D) branes [13], which correspond on the heterotic side to non perturbative states.

A second, far more surprising conclusion was reached in studying the strong coupling limit of the ten dimensional Type IIA string. It was already known that a simple truncation of eleven dimensional supergravity [44] on a circle of radius R_{11} gives the Type IIA supergravity in 10d, and that the Type IIA string coupling λ is related to the radius by [45]

$$M_{11}R_{11} = \lambda^{2/3} . \quad (7)$$

On the other hand, if we consider Kaluza-Klein masses of the compactified 11d supergravity and map them in Type IIA string units, we find

$$m_n = \frac{n}{R_{11}} \leftrightarrow m_n = \frac{n}{\lambda} M_{IIA} . \quad (8)$$

Therefore, on Type IIA side, they can be interpreted as non perturbative, and, with a bit more effort, BPS D0 brane states. The natural conclusion is that in the strong coupling limit $\lambda \rightarrow \infty$ of the Type IIA string, a new dimension reveals itself ($R_{11} \rightarrow \infty$ using (7)) and the low energy theory becomes the uncompactified 11d supergravity [48], [45]! As there is no known quantum theory whose low energy limit describes the 11d supergravity, a new name was invented for this underlying structure, the M-theory [12].

Soon after, Horava and Witten gave convincing arguments that the 11d supergravity compactified on a line segment S^1/Z_2 (or, equivalently, on a circle with opposite points identified) should describe the strong coupling limit of the $E_8 \times E_8$ heterotic string [27]. They argued that the two gauge factors sit at the ends of the interval, very much like the gauge quantum numbers of open strings are sitting at their ends. The basic argument is that only half (one Majorana-Weyl) of the original (Majorana) 11d gravitino lives on the boundary. This would produce gravitational anomalies unless 248 new Majorana-Weyl fermions appear at each end. This is exactly the dimension of the gauge group E_8 .

The compactification pattern of this theory down to 4d is different according to the relative value of the eleventh radius compared to the other radii, that are denoted collectively R in the following. Assuming for simplicity an isotropic compact space, there are two distinct compactification patterns

$$\begin{aligned} R_{11} < R &: 11d \rightarrow 10d \rightarrow 4d, \\ R_{11} > R &: 11d \rightarrow 5d \rightarrow 4d. \end{aligned} \tag{9}$$

In the strong coupling limit $R_{11} > R$, there is therefore an energy range where the spacetime is effectively five dimensional.

Finally, let us notice that in ten dimensions the Type IIB string is conjectured to be self-dual in the sense of an $SL(2, Z)$ strong-weak coupling S-duality. Moreover, the $SO(32)$ heterotic string compactified on a circle of radius R is T-dual to $E_8 \times E_8$ heterotic string compactified on a circle of radius $1/R$ and similarly Type IIA and Type IIB strings are T-dual to each other. By combining all the above information one can build a whole web of dualities, which becomes richer and richer when new space dimensions are compactified [49].

In the light of the new picture described above, let us see what changes in the strong coupling regime and let us investigate whether, for a string scale of the order of the GUT scale, the gauge unification problem has a natural solution in a region of *large* string coupling constant. The behavior is completely different depending on whether one considers the $SO(32)$ or the $E_8 \times E_8$ heterotic string.

Let us first consider the strongly-coupled $SO(32)$ heterotic string, equivalent to the weakly-coupled Type I string. We repeat the above discussion, using the Type I dilaton ϕ_I , metric g_I , and scalar curvature R_I . The analog of (2) is

$$L_{eff} = \int d^{10}x \sqrt{g_I} \left(e^{-2\phi_I} \frac{4}{(\alpha')^4} R_I - e^{-\phi_I} \frac{1}{(\alpha')^3} \text{tr} F^2 + \dots \right). \tag{10}$$

Contrary to the heterotic string case, the gravitational and gauge actions multiply different

functions of ϕ_I , $e^{-2\phi_I}$ and $e^{-\phi_I}$, since the first is generated by a world-sheet path integral on the sphere, while the second arises from the disk. The analog of (3) is then

$$L_{eff} = \int d^4x \sqrt{g_I} V \left(\frac{4e^{-2\phi_I}}{(\alpha')^4} R - \frac{e^{-\phi_I}}{(\alpha')^3} \text{tr} F^2 + \dots \right) . \quad (11)$$

The 4d quantities can be expressed as

$$M_I = \left(\frac{2}{\alpha_{GUT}^2 M_P^2} \right)^{1/4} V^{-1/4} , \quad \lambda_I = 4\alpha_{GUT} M_I^6 V . \quad (12)$$

Hence

$$M_I = \left(\frac{\alpha_{GUT} \lambda_I}{8} \right)^{1/2} M_P , \quad (13)$$

showing that after taking α_{GUT} from experiment one can make M_I as small as one wishes simply by taking e^{ϕ_I} to be small, that is, by taking the Type I superstring to be weakly coupled¹. In particular, as mentioned in the Introduction, M_I can be lowered down to the weak scale [35]. In this case the unification picture is completely different [37, 50], as we will see in the following sections.

We will now argue that the $E_8 \times E_8$ heterotic string has a similar strong coupling behavior: one retains the standard GUT relations among the gauge couplings, but loosing the prediction for Newton's constant, which can thus be considerably below the weak coupling bound.

At strong coupling, the ten-dimensional $E_8 \times E_8$ heterotic string becomes M -theory on $R^{10} \times S^1/Z_2$ [27]. The gravitational field propagates in the bulk of the eleventh dimension, while the $E_8 \times E_8$ gauge fields live at the Z_2 fixed points 0 and πR_{11} . We write M^{11} for $R^{10} \times S^1$ and M_i^{10} , $i = 1, 2$ for the two fixed (hyper)planes. The gauge and gravitational kinetic energies take the form

$$L = \frac{1}{2\kappa_{11}^2} \int_{M^{11}} d^{11}x \sqrt{g} R - \sum_i \frac{3^{1/3}}{4\pi(2\pi\kappa_{11}^2)^{2/3}} \int_{M_i^{10}} d^{10}x \sqrt{g} \text{tr} F_i^2 , \quad (14)$$

¹In this case, however, $M_I^6 V \ll 1$ and a better physical picture is obtained by performing T-dualities, thus generating lower-dimensional branes.

where κ_{11} is here the eleven-dimensional gravitational coupling and F_i , for $i = 1, 2$, is the field strength of the i^{th} E_8 , which propagates on the fixed plane M_i^{10} .

Now compactify to four/five dimensions on a compact manifold whose volume (in the eleven-dimensional metric, from now on) is V . Let S^1 have a radius R_{11} , or a circumference $2\pi R_{11}$, and define the eleven dimensional scale $M_{11} = 2\pi(4\pi\kappa_{11}^2)^{-1/9}$. Upon reducing (14) down to 4d, one can express M_{11} and R_{11} in terms of four-dimensional parameters

$$M_{11} = (2\alpha_{GUT}V)^{-1/6}, R_{11}^{-1} = \left(\frac{2}{\alpha_{GUT}}\right)^{3/2} M_P^{-2} V^{-1/2}. \quad (15)$$

From the first relation we find that $M_{11} \sim M_{GUT}$, and therefore the Horava-Witten theory can accomodate a traditional MSSM unification-type scenario with fundamental scale $M_{11} \sim 10^{16}$ GeV. The second one, for $V = M_{GUT}^{-6}$, gives $R_{11}^{-1} \sim 10^{13} - 10^{15}$ GeV. This is again a sensible result, since R_{11} has to be large compared to the eleven-dimensional Planck scale in order to have a reliable field-theory description of the theory.

3. Building blocks for Type I strings

Type I strings describe the dynamics of open and closed superstrings. Denoting by $0 \leq \sigma \leq \pi$ the coordinate describing the open string at a given time, the two ends $\sigma = 0, \pi$ contain the gauge group (Chan-Paton) degrees of freedom and the corresponding charged matter fields. The open string quantum states can be conveniently described by matrices

$$|k; a \rangle = \sum_{i,j=1}^N \lambda_{i,j}^a |k; i, j \rangle, \quad (16)$$

where $i, j = 1 \cdots N$ denote Chan-Paton indices and k other internal quantum numbers. At the ends of open strings, we must add boundary conditions, which for string coordinates can be of two types

$$\frac{\partial X^\mu}{\partial \sigma} \Big|_{\sigma=0,\pi} = 0 \quad , \quad (N) \quad , \quad X^\mu \Big|_{\sigma=0,\pi} = \text{cst} \quad , \quad (D) \quad , \quad (17)$$

where the two different possibilities denote the Neumann (N) and Dirichlet (D) strings. As will be explained later on, even if we start with a theory containing only Neumann strings, the Dirichlet strings can arise after performing various T-duality operations or on orbifolds containing Z_2 -type elements. Since by joining two open strings one can create a closed string, propagation of closed strings must be added for consistency. The corresponding quantum fluctuations produce the closed (gravitational-type) spectrum of the theory, neutral under the Chan-Paton gauge group, that always contains the gravitational (super)multiplet. The string oscillators are defined as Fourier modes of the string coordinates. For closed coordinates, the expansion reads

$$X_c^\mu = x^\mu + 2\alpha' p^\mu \tau + \frac{i}{2} \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \left[\alpha_n^\mu e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)} \right] . \quad (18)$$

The usual canonical quantization gives the commutators for the left movers $[\alpha_m^\mu, \alpha_n^\mu] = m\delta_{m+n}\eta^{\mu\nu}$ and similarly for the right movers. For open strings with Neumann boundary conditions, for example, the oscillator expansion reads

$$X_o^\mu = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \left[\alpha_n^\mu e^{-in\tau} \cos n\sigma \right] . \quad (19)$$

Type I can be seen as a projection (or *orientifold*) of Type IIB theory, obtained by projecting the Type IIB spectrum by the involution Ω , exchanging the left and right closed oscillators $\alpha_m^\mu, \tilde{\alpha}_m^\mu$ and acting on the open-strings ones by phases

$$\text{closed} : \Omega \quad : \quad \alpha_m^\mu \leftrightarrow \tilde{\alpha}_m^\mu \quad , \quad \text{open} : \Omega \quad : \quad \alpha_m^\mu \rightarrow \pm(-1)^m \alpha_m^\mu . \quad (20)$$

In addition, Ω acts on the zero-modes (compactification lattice) of closed strings by interchanging left and right momenta $\mathbf{p}_L \leftrightarrow \mathbf{p}_R$.

The parent Type IIB string contains D(-1), D1, D3, D5, D7 (and D9) branes, coupling electrically or magnetically to the various RR forms present in the massless spectrum. Out of them, the D1, D5 and D9 branes are invariant under Ω and therefore are present in the Type I theory, as (sub)spaces on which open string ends can terminate. In some sense,

open strings can be considered as twisted states of the Ω involution [8], in analogy with twisted states in orbifold compactifications of closed strings.

The perturbative, topological expansion in Type I strings involves two-dimensional surfaces with holes h , boundaries b and crosscaps c . Each surface has an associated factor $\lambda_I^{-\chi}$, where

$$\chi = 2 - 2h - b - c \quad (21)$$

is the Euler genus of the corresponding surface. Tree-level diagrams include, in addition to the sphere with genus $\chi = 2$, the disk with one boundary $\chi = 1$, where open string vertex operators can be attached, and the projective plane RP^2 with one crosscap ($\chi = 1$). One-loop diagrams include, in addition to the usual torus \mathcal{T} with one handle, the Klein bottle \mathcal{K} with two crosscaps, the annulus \mathcal{A} with two boundaries and the Möbius \mathcal{M} with one boundary and one crosscap, all of them having $\chi = 0$. The last two diagrams allow the propagation of open strings with Chan-Paton charges $|k; ij\rangle$ in the annulus and $|k; ii\rangle$ in the Möbius, containing the gauge group and the charged matter degrees of freedom. On the other hand, the torus and the Klein bottle describe the propagation of closed string degrees of freedom.

One-loop string diagrams may be constructed as generalizations of the one-loop vacuum energy in field-theory. In d noncompact dimensions, the vacuum energy contribution of a real boson of mass m is

$$\begin{aligned} \Gamma &= \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \ln(p^2 + m^2) = -\frac{1}{2} \int_0^\infty \frac{dt}{t} \int \frac{d^d p}{(2\pi)^d} e^{-(p^2 + m^2)t} \\ &= -\frac{1}{2(4\pi)^{d/2}} \int_0^\infty \frac{dt}{t^{1+d/2}} e^{-tm^2}, \end{aligned} \quad (22)$$

where we introduced a Schwinger proper-time parameter through the identity

$$\ln \frac{A}{B} = - \int_0^\infty \frac{dt}{t} (e^{-tA} - e^{-tB}) \quad (23)$$

and where we also neglected in (22) an (infinite) irrelevant mass-independent term. The result (22) readily generalizes to the case of more particles in the loop with mass operator

m and different spin, as

$$\Gamma = -\frac{1}{2(4\pi)^{d/2}} \text{Str} \int_0^\infty \frac{dt}{t^{1+d/2}} e^{-tm^2} , \quad (24)$$

where Str takes into account the multiplicities of particles and their spin and reduces in 4d to the usual definition $\text{Str } m^{2k} = \sum_J (-1)^{2J} (2J+1) \text{tr } m_J^{2k}$, where m_J denotes the mass matrix of particles of spin J .

The generalization of (24) to the Type IIB torus partition function in d noncompact (and $10-d$ compact) dimensions is (keeping only internal metric moduli here for simplicity)

$$\begin{aligned} \mathcal{T} = \text{Tr} \frac{1+(-1)^G}{2} \frac{1+(-1)^{\bar{G}}}{2} \mathcal{P} q^{L_0} \bar{q}^{\bar{L}_0} = \\ \frac{1}{(4\pi^2\alpha')^{\frac{d}{2}}} \sum_{rs} X_{rs} \int_F \frac{d^2\tau}{(\text{Im } \tau)^{1+\frac{d}{2}}} \chi_r(\tau) \chi_s(\bar{\tau}) \Gamma_{rs}^{(10-d,10-d)}(\tau, \bar{\tau}, g_{i\bar{j}}) , \end{aligned} \quad (25)$$

where $q = \exp(2\pi i\tau)$ and τ is the modular parameter of the torus, L_0, \bar{L}_0 are Virasoro operators for the left and the right movers, $(-1)^G ((-1)^{\bar{G}})$ is the world-sheet left (right) fermion number implementing the GSO projection and \mathcal{P} an operator needed in orbifold compactifications (see Section 5) in order to project onto physical states. In (25), the χ 's are a set of modular functions of the underlying conformal field theory, $\Gamma_{rs}^{(10-d,10-d)}$ is the contribution from the compactification lattice depending on the compact metric components $g_{i\bar{j}}$ and X is a matrix of integers. The integral in (25) is performed over the fundamental region

$$F : \text{Im}\tau \geq 0 , \quad -\frac{1}{2} \leq \tau_1 \leq \frac{1}{2} , \quad |\tau| \geq 1 , \quad (26)$$

and the $\text{Im } \tau$ factors come from integrating over noncompact momenta. The typical form of the characters is

$$\chi_r = q^{h_r - \frac{c}{24}} \sum_{n=0}^{\infty} d_n^r q^n , \quad (27)$$

where h_r is the conformal weight, c is the central charge of the conformal field theory and the d_n^r are positive integers.

Let us start with a brief review of the algorithm used in the following. This was introduced in [9, 10], and developed further in [54]. The starting point consists in adding

to the (halved) torus amplitude the Klein-bottle \mathcal{K} . This completes the projection induced by Ω , and is a linear combination of the diagonal contributions to the torus amplitude, with argument $q\bar{q}$. Then one obtains ²

$$\mathcal{K} = Tr \frac{\Omega}{2} \frac{1 + (-1)^G}{2} \mathcal{P} e^{-4\pi\tau_2 L_0} = \frac{1}{2(4\pi^2\alpha')^{\frac{d}{2}}} \int_0^\infty \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}} \sum_r X_{rr} \chi_r(2i\tau_2) \Gamma_{\mathcal{K},r}^{(10-d)}(i\tau_2, g_{i\bar{j}}) , \quad (28)$$

with τ_2 the proper time for the closed string and $\Gamma_{\mathcal{K},r}^{(10-d)}(i\tau_2, g_{i\bar{j}})$ the contribution of the compactification lattice. In order to identify the corresponding open sector, it is useful to perform the S modular transformation induced by

$$\mathcal{K} : \quad 2\tau_2 \quad \xrightarrow{s} \quad \frac{1}{2\tau_2} \equiv l , \quad (29)$$

thus turning the direct-channel Klein-bottle amplitude \mathcal{K} into the transverse-channel amplitude. The latter describes the propagation of the closed spectrum on a cylinder of length l terminating at two crosscaps³, and has the generic form

$$\mathcal{K} = \frac{1}{2(4\pi^2\alpha')^{\frac{d}{2}}} \int_0^\infty dl \sum_r \Gamma_r^2 \chi_r(il) \tilde{\Gamma}_{\mathcal{K},r}^{(10-d)}(il, g_{i\bar{j}}) \equiv \frac{1}{2(4\pi^2)^{\frac{d}{2}}\alpha'^{\frac{5}{2}}} \int_0^\infty dl \tilde{\mathcal{K}} , \quad (30)$$

where $\tilde{\Gamma}_{\mathcal{K},r}^{(10-d)}(il, g_{i\bar{j}})$ is the Poisson transform of $\Gamma_{\mathcal{K},r}$ and the coefficients Γ_r can be related to the one-point functions of the closed-string fields in the presence of a crosscap. Alternatively, in a spacetime language, the Ω involution has fixed (hyper)surfaces called orientifold (O) planes, carrying RR charge. The Klein bottle amplitude is then interpreted as describing the closed string propagation starting and ending on orientifold (O) planes. Since the modulus of the Klein amplitude is $0 \leq \tau_2 < \infty$, the τ_2 integral is not cut in the ultraviolet (UV) and is generically UV divergent. Physically, this divergence is related to

²As discussed in [54], in general one has the option of modifying eq. (28), altering X_{ii} by signs ϵ_i . These turn sectors symmetrized under left-right interchange into antisymmetrized ones, and vice-versa, and are in general constrained by compatibility with the fusion rules. This freedom, which has the spacetime interpretation of flipping the RR charge of some orientifold planes, will turn out to be crucial later on.

³The crosscap, or real projective plane, is a non-orientable surface that may be defined starting from a 2-sphere and identifying antipodal points.

the presence of an uncanceled RR flux from the O planes, which asks for the introduction of D branes and corresponding open strings. It will be important later on to distinguish between several types of O-planes. First of all, in supersymmetric models there are O_+ planes carrying negative RR charge and O_- planes carrying positive RR charge and also flipped NS-NS couplings, in order to preserve supersymmetry. In nonsupersymmetric models, there can exist \bar{O}_+ planes with flipped RR charge compared to their supersymmetric O_+ cousins, but with the same NS-NS couplings, therefore breaking supersymmetry. Analogously, we can define \bar{O}_- planes, starting from O_- planes and flipping only the RR charge. We will exemplify later on in detail the couplings of these four different types of O-planes to supergravity fields in different models.

The open strings may be deduced from the closed-string spectrum in a similar fashion. A very important property of one-loop open string amplitudes is that they all have a dual interpretation as tree-level closed string propagation (see Figure 1). First, the direct-channel annulus amplitude may be deduced from the transverse-channel boundary-to-boundary amplitude. This has the general form [9] (see also [11])

$$\mathcal{A} = \frac{1}{(8\pi^2\alpha')^{\frac{d}{2}}} \int_0^\infty dl \sum_r B_r^2 \chi_r(il) \tilde{\Gamma}_{\mathcal{A},r}^{(10-d)}(il, g_{i\bar{j}}) \equiv \frac{1}{(8\pi^2)^{\frac{d}{2}}\alpha'^{\frac{5}{2}}} \int_0^\infty dl \tilde{\mathcal{A}}, \quad (31)$$

where the coefficients B_r can be related to the one-point functions of closed-string fields on the disk and on the RP^2 crosscap. In a spacetime interpretation, the annulus amplitudes describe open strings with ends stuck on D branes. The relevant S modular transformation now maps the closed string proper time l on the tube to the open-string proper time t on the annulus, according to

$$\mathcal{A}: \quad l \quad \xrightarrow{s} \quad \frac{1}{l} \equiv \frac{t}{2}. \quad (32)$$

The direct-channel annulus amplitude then takes the form

$$\mathcal{A} = \frac{1}{2} Tr \frac{1 + (-1)^G}{2} \mathcal{P} e^{-\pi t L_0} = \frac{1}{2(8\pi^2\alpha')^{\frac{d}{2}}} \int_0^\infty \frac{dt}{t^{1+\frac{d}{2}}} \sum_{r,a,b} A_{ab}^r n_a n_b \chi_r\left(\frac{it}{2}\right) \Gamma_{\mathcal{A},r}^{(10-d)}\left(\frac{it}{2}, g_{i\bar{j}}\right), \quad (33)$$

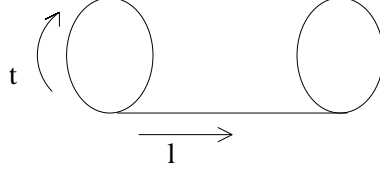


Figure 1: The annulus amplitude has a dual interpretation of one-loop open string propagation with vertical time t and tree-level closed string propagation with horizontal time l .

where L_0 in (33) is the Virasoro operator in the open sector, the n 's are integers that have the interpretation of Chan-Paton multiplicities for the boundaries (D branes) and the A^r are a set of matrices with integer elements. These matrices are obtained solving diophantine equations determined by the condition that the modular transform of eq. (33) involves only integer coefficients, while the Chan-Paton multiplicities arise as free parameters of the solution. Supersymmetric models contain only D-branes, i.e. objects carrying positive RR charges. Nonsupersymmetric models ask generically also for *antibranes*, objects carrying negative RR charges but with NS-NS couplings identical to those of branes.

Finally, the transverse-channel Möbius amplitude $\tilde{\mathcal{M}}$ describes the propagation of closed strings between D branes and O planes (or boundaries and crosscaps, in worldsheet language), and is determined by factorization from $\tilde{\mathcal{K}}$ and $\tilde{\mathcal{A}}$. It contains the characters common to the two expressions, with coefficients that are geometric means of those present in $\tilde{\mathcal{K}}$ and $\tilde{\mathcal{A}}$ [9], [10]. Thus

$$\mathcal{M} = -2 \frac{1}{(8\pi^2 \alpha')^{\frac{d}{2}}} \int_0^\infty dl \sum_r B_r \Gamma_r \hat{\chi}_r \left(il + \frac{1}{2} \right) \tilde{\Gamma}_{\mathcal{M},r}^{(10-d)}(il, g_{i\bar{j}}) \equiv \frac{1}{(8\pi^2)^{\frac{d}{2\alpha'^5}}} \int_0^\infty dl \tilde{\mathcal{M}}, \quad (34)$$

where the hatted characters form a real basis and are obtained by the redefinitions

$$\hat{\chi}_r \left(il + \frac{1}{2} \right) = e^{-i\pi h_r} \chi_r \left(il + \frac{1}{2} \right). \quad (35)$$

The direct-channel Möbius amplitude can then be related to $\tilde{\mathcal{M}}$ by a modular P transfor-

mation and by the redefinition (35)

$$\mathcal{M} : \quad \frac{it}{2} + \frac{1}{2} \quad \xrightarrow{P} \quad \frac{i}{2t} + \frac{1}{2} \equiv il + \frac{1}{2} . \quad (36)$$

This is realized on the hatted characters by the sequence $P = T^{1/2} S T^2 S T^{1/2}$, with S the matrix that implements the transformation $\tau \rightarrow -1/\tau$ and T the diagonal matrix that implements the transformation $\tau \rightarrow \tau + 1$. The direct-channel Möbius amplitude then takes the form

$$\begin{aligned} \mathcal{M} = & Tr \frac{\Omega}{2} \frac{1 + (-1)^G}{2} \mathcal{P} (-e^{-\pi t})^{L_0} = \\ & - \frac{1}{2(8\pi^2 \alpha')^{\frac{d}{2}}} \int_0^\infty \frac{dt}{t^{1+\frac{d}{2}}} \sum_{r,a} M_a^r n_a \hat{\chi}_r \left(\frac{it}{2} + \frac{1}{2} \right) \Gamma_{\mathcal{M},r}^{(10-d)} \left(\frac{it}{2}, g_{i\bar{j}} \right) , \end{aligned} \quad (37)$$

where by consistency the integer coefficients M_a^r satisfy constraints [55] that make \mathcal{M} the Ω projection of \mathcal{A} . The full one-loop vacuum amplitude is

$$\int \left(\frac{1}{2} \mathcal{T}(\tau, \bar{\tau}) + \mathcal{K}(2i\tau_2) + \mathcal{A}\left(\frac{it}{2}\right) + \mathcal{M}\left(\frac{it}{2} + \frac{1}{2}\right) \right) , \quad (38)$$

where the different measures of integration are left implicit. In the remainder of this paper, we shall often omit the dependence on world-sheet modular parameters.

It is often convenient for a spacetime particle interpretation to write the partition functions with the help of $SO(2n)$ characters

$$\begin{aligned} O_{2n} &= \frac{1}{2\eta^n} (\theta_3^n + \theta_4^n) , & V_{2n} &= \frac{1}{2\eta^n} (\theta_3^n - \theta_4^n) , \\ S_{2n} &= \frac{1}{2\eta^n} (\theta_2^n + i^n \theta_1^n) , & C_{2n} &= \frac{1}{2\eta^n} (\theta_2^n - i^n \theta_1^n) , \end{aligned} \quad (39)$$

where the θ_i are the four Jacobi theta-functions with (half)integer characteristics. In a spacetime interpretation, at the lowest level O_{2n} represents a scalar, V_{2n} represents a vector, while S_{2n} , C_{2n} represent spinors of opposite chiralities. In order to link the direct and transverse channels, one needs the transformation matrices S and P for the level-one $SO(2n)$ characters (39). These may be simply deduced from the corresponding transfor-

mation properties of the Jacobi theta functions, and are

$$S_{(2n)} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix}, \quad P_{(2n)} = \begin{pmatrix} c & s & 0 & 0 \\ s & -c & 0 & 0 \\ 0 & 0 & \zeta c & i\zeta s \\ 0 & 0 & i\zeta s & \zeta c \end{pmatrix}, \quad (40)$$

where $c = \cos(n\pi/4)$, $s = \sin(n\pi/4)$ and $\zeta = e^{-in\pi/4}$ [10].

The absence of UV divergences ($l \rightarrow \infty$ limit) in the above amplitudes asks for constraints on the Chan-Paton factors, called *tadpole consistency conditions* [53]. They are equivalent to the absence of tree-level one-point functions for some closed string fields and ensure that the total RR charge in the theory is zero. In the notations used here, they read

$$B_r = \Gamma_r, \quad (41)$$

and generically determine the Chan-Paton multiplicity, that in ten dimensions equals $N = 32$. The tadpoles for RR fields can be related [53] to inconsistencies in the field equations of RR forms (often reflected in the presence of gauge and gravitational anomalies). Indeed, D branes and O planes are electric and magnetic sources for RR forms. The Bianchi identities and field equations for a form of order n then read (in the language of differential forms)

$$dH_{n+1} = *J_{8-n}, \quad d * H_{n+1} = *J_n, \quad (42)$$

where the subscript on the electric and magnetic sources denotes their rank. The field equations are globally consistent if

$$\int_{C_m} *J_{10-m} = 0, \quad (43)$$

for all closed (sub)manifolds C_m . In particular, in a compact space the RR flux must be zero, and this gives nontrivial constraints on the spectrum of D branes in the theory.

The situation is different for NS-NS tadpoles. Indeed, suppose there is a dilaton tadpole, of the type $\exp(-\Phi)$ in the string frame, generated by the presence of (anti)brane-

(anti)orientifold Dp-Op systems. The dilaton classical field equation reads

$$\partial_A(\sqrt{g} g^{AB} \partial_B \Phi) = \sum_i \alpha_i \sqrt{g} e^{\frac{(p-3)\Phi}{4}} \delta^{(9-p)}(y - y_i) , \quad (44)$$

where $A, B = 1 \cdots 10$ and y_i denote the position of the brane-orientifold planes in the space transverse to the brane. The uncanceled dilaton tadpole means explicitly

$$\sum_i \alpha_i \neq 0 , \text{ while } \sum_i \alpha_i \int_{\mathcal{C}} \sqrt{g} e^{\frac{(p-3)\Phi}{4}} \delta^{(9-p)}(y - y_i) = 0 . \quad (45)$$

The first inequality means that, around the *flat vacuum*, the r.h.s. source in (44) does not integrate to zero and violates the integrability condition coming from the l.h.s. of (44). As stressed in [56], however, this simply means that the real background is *not* the flat background, but a curved one. This explains the second equality in (45), where \mathcal{C} is any closed curve or (hyper)surface in the internal space. An explicit example of such a Type I background was recently given in [58].

In order to describe some simple Type I examples, let us consider two 10d orientifolds of Type IIB.

i) Supersymmetric SO(32)

The Type IIB torus amplitude reads

$$\mathcal{T} = \frac{1}{(4\pi^2 \alpha')^5} \int_F \frac{d^2 \tau}{(\text{Im } \tau)^6} |(V_8 - S_8) \frac{1}{\eta^8}|^2 , \quad (46)$$

in terms of the characters introduced in (39). In (46), the characters $V_8 - S_8$ describe the contribution of the (worldsheet, left and right) fermionic coordinates $(\Psi^\mu, \tilde{\Psi}^\mu)$ to the partition function. Moreover, $1/\eta^8$ denotes the contribution of the eight transverse bosons X^μ , where η is the Dedekind modular function defined in eq. (243) of the Appendix. The corresponding Klein bottle amplitude is

$$\mathcal{K} = \frac{1}{2(4\pi^2 \alpha')^5} \int_0^\infty \frac{d\tau_2}{\tau_2^6} (V_8 - S_8) \frac{1}{\eta^8} . \quad (47)$$

It symmetrizes the NS-NS states and antisymmetrizes the RR states. In particular, the NS-NS antisymmetric tensor is projected out of the spectrum (still, quantized parts of it can

consistently be introduced [51]), while the RR antisymmetric tensor survives, a general feature in Type I models.

The annulus and Möbius amplitudes in the open string channel read

$$\begin{aligned}\mathcal{A} &= \frac{N^2}{2} \frac{1}{(8\pi^2\alpha')^5} \int_0^\infty \frac{dt}{t^6} (V_8 - S_8) \frac{1}{\eta^8} , \\ \mathcal{M} &= -\frac{N}{2} \frac{1}{(8\pi^2\alpha')^5} \int_0^\infty \frac{dt}{t^6} (V_8 - S_8) \frac{1}{\eta^8} ,\end{aligned}\tag{48}$$

where N is the Chan-Paton index. In order to find the massless spectrum, we expand them in powers of the modular parameter q and retain the constant piece, obtaining

$$\mathcal{A}_0 + \mathcal{M}_0 \sim \frac{N(N-1)}{2} \int_0^\infty \frac{dt}{t^6} \times (8-8) ,\tag{49}$$

where the $(8-8)$ terms come from the vector V_8 and the spinor S_8 , respectively. The massless spectrum is therefore supersymmetric, and consists of 10d vectors and Weyl spinors in the adjoint representation of the gauge group $SO(N)$. The dimension of the group is fixed by looking at the divergent (tadpole) piece of the amplitudes in the transverse (closed-string) channel

$$\mathcal{K} + \mathcal{A} + \mathcal{M} = \frac{1}{2} \frac{1}{(8\pi^2\alpha')^5} \int_0^\infty dl \left(32 + \frac{N^2}{32} - 2N \right) \times (8-8) + \dots ,\tag{50}$$

where the two equal terms $8-8$ come from the NS-NS and RR massless closed string states and \dots denote exchanges of massive closed states, with no associated IR divergences. Even if the RR and NS-NS divergent pieces cancel each other, consistency of the theory requires cancelling each independently, as they reflect the existence of different couplings. This requires that $N = 32$ and therefore determines the gauge group $SO(32)$. The geometric interpretation of this model is that it contains 32 D9 branes and 32 O9₊ planes, that carry RR charge under an unphysical RR 10-form A_{10} . The effective action contains the bosonic terms

$$S = \int d^{10}x \{ \sqrt{g} \mathcal{L}_{SUGRA} - (N-32)(\sqrt{g}e^{-\Phi} + A_{10}) \} + \dots ,\tag{51}$$

clearly displaying the interaction of closed fields g_{AB}, Φ, A_{10} with the D9 branes and the O9 planes in the model.

ii) Nonsupersymmetric $USp(32)$

As already explained, there is an important difference between tadpoles of RR closed fields and tadpoles of NS-NS closed fields. While the first signal an internal inconsistency of the theory and must therefore always be cancelled, the latter ask for a background redefinition and remove flat directions, producing potentials for the corresponding fields and leading actually to consistent models [56]. The difference between RR and NS-NS tadpoles turns out to play an important role in (some) models with broken supersymmetry. Indeed, there is another consistent model in 10d described by the same closed spectrum (46)-(47), but with a nonsupersymmetric open spectrum described by the Chan-Paton charge N . The open string partition functions are [57]

$$\begin{aligned}\mathcal{A} &= \frac{N^2}{2} \frac{1}{(8\pi^2\alpha')^5} \int_0^\infty \frac{dt}{t^6} (V_8 - S_8) \frac{1}{\eta^8} , \\ \mathcal{M} &= \frac{N}{2} \frac{1}{(8\pi^2\alpha')^5} \int_0^\infty \frac{dt}{t^6} (V_8 + S_8) \frac{1}{\eta^8} .\end{aligned}\tag{52}$$

From the closed string viewpoint, V_8 describes the NS-NS sector (more precisely, the dilaton) and S_8 the RR sector. The tadpole conditions here read

$$\mathcal{K} + \mathcal{A} + \mathcal{M} = \frac{1}{2} \frac{1}{(8\pi^2\alpha')^5} \int_0^\infty dl \{ (N+32)^2 \times 8 - (N-32)^2 \times 8 \} + \dots .\tag{53}$$

It is therefore clear that we can set to zero the RR tadpole choosing $N = 32$, but we are forced to live with a dilaton tadpole. The resulting spectrum is nonsupersymmetric and contains the vectors of the gauge group $USp(32)$ and fermions in the antisymmetric (reducible) representation. However, the spectrum is free of gauge and gravitational anomalies, and therefore the model appears to be consistent. This model contains 32 $\bar{D}9$ branes and 32 O9₋ planes, such that the total RR charge is zero but NS-NS tadpoles are present, signaling the breaking of supersymmetry. The effective action contains the bosonic

terms

$$S = \int d^{10}x \{ \sqrt{g} \mathcal{L}_{SUGRA} - (N + 32) \sqrt{g} e^{-\Phi} + (N - 32) A_{10} \} + \dots . \quad (54)$$

Notice in (54) the peculiar couplings of the dilaton and the 10-form to antibranes/ O_- planes, in agreement with the general properties displayed earlier. Indeed, the coupling to the ten-form is similar to the supersymmetric one (51), modulo the overall sign reflecting the flipped RR charge of antibranes and O_- planes compared to branes and O_+ planes. The coupling to the dilaton reflects that antibranes couple to NS-NS fields in the same way as branes, while O_- planes couple with a flipped sign compared to O_+ planes.

The NS-NS tadpoles generate scalar potentials for the corresponding (closed-string) fields, in our case the (10d) dilaton. The dilaton potential reads

$$V \sim (N + 32) e^{-\Phi} , \quad (55)$$

and in the Einstein frame is proportional to $(N + 32) \exp(3\Phi/2)$. It has therefore the (usual) runaway behaviour towards zero string coupling, a feature which is common to all perturbative constructions. However, other NS-NS fields can be given more complicated potentials and can be stabilized in appropriate compactifications of Type I strings with brane-antibrane systems, as we will see later on. Due to the dilaton tadpole, the background of this model is not the 10d Minkowski space. However, it was shown in [58] that a background with $SO(9)$ Poincare symmetry can be explicitly found, therefore curing the NS-NS tadpole problem. In this background, the tenth dimension is spontaneously compactified and the geometry is $R^9 \times S^1/Z_2$, with localized gravity.

There is another way to see that in 10d the only possible gauge groups are orthogonal and symplectic. Indeed, the massless gauge bosons are represented as $\lambda \alpha_{-1}^A |0\rangle$, where λ is the matrix describing the Chan-Paton charges defined in (16), of size $N \times N$. The orientifold involution Ω which squares to one can have a nontrivial action on the matrix λ

$$\Omega \quad : \quad \lambda \rightarrow -\gamma_{\Omega} \lambda^T \gamma_{\Omega}^{-1} , \quad (56)$$

where λ^T denotes the transpose of the matrix λ . The action of Ω squares to one if $\gamma_\Omega(\gamma_\Omega^{-1})^T = \pm I$, where I is the identity matrix. Then the gauge bosons are invariant under Ω if [42]

a) $\gamma_\Omega = \gamma_\Omega^T = I$, implying $\lambda = -\lambda^T$ and the gauge group is $SO(N)$.

b) $\gamma_\Omega = -\gamma_\Omega^T$, implying $\lambda = \lambda^T$ and the gauge group $USp(N)$.

The difference between the supersymmetric $SO(32)$ and nonsupersymmetric $USp(32)$ model described previously is that in the supersymmetric case Ω acted in the same way on NS-NS and RR states in the (transverse channel) Möbius, while in the nonsupersymmetric case the action was $\Omega = 1$ for NS-NS states and $\Omega = -1$ for RR states. Both possibilities are however consistent with the rules described at the beginning of this section, namely particle interpretation and factorization. We will interpret later the first model as containing 32 D9 branes and the second one as containing 32 $D\bar{9}$ (anti)branes, where by definition antibranes have reversed RR charge compared to the corresponding branes. The $USp(32)$ model is interpreted as containing 32 $O9_-$ planes of positive RR charge (instead of the negative charged $O9_+$ of the supersymmetric case), asking for 32 $D\bar{9}$ (anti)branes in the open sector. The only change occurs in the Möbius amplitude, that describes strings stretched between (anti)branes and orientifold planes.

iii) Models with local tadpole cancellation

As we have seen in the previous models, UV divergences in the open spectrum are related to tadpoles of massless closed fields exchanged by the branes. Let us now compactify one dimension (the discussion easily generalizes to more compactified dimensions). The closed string fields have in this case a tower of winding states of mass nRM_I^2 that give no additional divergences. However, in the limit $R \rightarrow 0$ all these states become massless and contribute new potential divergences. For example, the supersymmetric $SO(32)$ model has, in the T-dual version ($R_\perp = 1/RM_I^2$), 32 D8 branes at the origin $y_\perp = 0$ and 32 orientifold O8 planes equally distributed between the two orientifold fixed planes $y_\perp = 0, \pi R_\perp$. The *global*

RR charge is indeed cancelled, however locally there are 16 units of RR charge at $y_\perp = 0$ and -16 at $y_\perp = \pi R_\perp$. Consequently, the dilaton has a variation along y_\perp and for $R_\perp \rightarrow \infty$, the theory encounters singularities [46]. Avoiding this pathology asks for a new condition, *local tadpole cancellation* or, equivalently the local cancellation of the RR charge. In the example at hand, the only Type I 9d model satisfying this condition is obtained putting, via a Wilson line, 16 D8 branes at the origin $y_\perp = 0$ and 16 D8 branes at $y_\perp = \pi R_\perp$, thus giving the gauge group $SO(16) \times SO(16)$.

This phenomenon manifests itself neatly in the one-loop vacuum amplitudes [19]. Indeed, let us compactify the 10d Type I string on a circle and let us introduce a Wilson line $W = (I_{n_1}, -I_{n_2})$, which breaks the gauge group $SO(32) \rightarrow SO(n_1) \times SO(n_2)$. The three relevant amplitudes read, in the direct channel,

$$\begin{aligned}\mathcal{K} &= \frac{1}{2(4\pi^2\alpha')^{9/2}} \int_0^\infty \frac{d\tau_2}{\tau_2^{11/2}} \sum_m P_m (V_8 - S_8) \frac{1}{\eta^8} , \\ \mathcal{A} &= \frac{1}{(8\pi^2\alpha')^{9/2}} \int_0^\infty \frac{dt}{t^{11/2}} (V_8 - S_8) \frac{1}{\eta^8} \left(\frac{n_1^2 + n_2^2}{2} \sum_m P_m + n_1 n_2 \sum_m P_{m+1/2} \right) , \\ \mathcal{M} &= -\frac{n_1 + n_2}{2} \frac{1}{(8\pi^2\alpha')^{9/2}} \int_0^\infty \frac{dt}{t^{11/2}} (V_8 - S_8) \frac{1}{\eta^8} \sum_m P_m ,\end{aligned}\tag{57}$$

where the half-integer powers of t and α' come from integrating over the nine noncompact momenta and the lattice summations are defined in the Appendix. The same amplitudes can be written (after S and P transformations), in the transverse channel,

$$\begin{aligned}\tilde{\mathcal{K}} &= \frac{2^5 R}{2\sqrt{2}} \sum_n W_{2n} (V_8 - S_8) \frac{1}{\eta^8} , \\ \tilde{\mathcal{A}} &= \frac{R}{2^5\sqrt{2}} (V_8 - S_8) \frac{1}{\eta^8} \left(\frac{n_1^2 + n_2^2}{2} \sum_n W_n + n_1 n_2 \sum_n (-1)^n W_n \right) , \\ \tilde{\mathcal{M}} &= -(n_1 + n_2) \frac{R}{\sqrt{2}} (V_8 - S_8) \frac{1}{\eta^8} \sum_n W_{2n} ,\end{aligned}\tag{58}$$

where the various numerical coefficients in (58) arise from the S transformations in the Klein bottle and annulus amplitude and after the P transformation in the Möbius amplitudes.

The sum of the three amplitudes

$$\tilde{\mathcal{K}} + \tilde{\mathcal{A}} + \tilde{\mathcal{M}} = \frac{R}{2\sqrt{2}}(V_8 - S_8) \frac{1}{\eta^8} \left\{ \left[32 + \frac{(n_1 + n_2)^2}{32} - 2(n_1 + n_2) \right] \sum_n W_{2n} + \frac{(n_1 - n_2)^2}{32} \sum_n W_{2n+1} \right\} \quad (59)$$

tells us that for an arbitrary radius the tadpole conditions coming from the massless states are

$$32 + \frac{(n_1 + n_2)^2}{32} - 2(n_1 + n_2) = 0 , \quad (60)$$

fixing the number of D9 branes $n_1 + n_2 = 32$. However, in the $R \rightarrow 0$ ($R_\perp \rightarrow \infty$) limit the odd winding states become massless too. Therefore the last term in (59) asks for $n_1 = n_2 = 16$ and the gauge group is $SO(16) \times SO(16)$, as anticipated. Models with local tadpole conditions are intimately related to M-theory compactifications since they allow, by a suitable identification of R_{11} with R_\perp , a well-defined strong coupling heterotic (M-theory) limit $R_\perp \rightarrow \infty$.

4. M-theory

The maximal supergravity theory (containing particles with spin less than or equal to two) was constructed long time ago by Cremmer, Julia and Scherk [44] in eleven dimensions and is unique. The field content consists of the irreducible 11d supergravity multiplet: the graviton g_{IJ} , the Majorana gravitino $\psi_{I\alpha}$ and the three-form C_{IJK} . The bosonic part of the action was found to be

$$L_{SUGRA} = \frac{1}{2\kappa_{11}^2} \int_{M^{11}} d^{11}x \sqrt{g} \left(R^{(11)} - \frac{1}{24} G_{IJKL} G^{IJKL} \right) - \frac{\sqrt{2}}{\kappa_{11}^2} \int_{M^{11}} d^{11}x C \wedge G \wedge G , \quad (61)$$

where $I, J, K, L = 1 \cdots 11$ and G is the field-strength of the three-form ($G = 6dC$ in form notation), where we use here and in the following the usual definition $A = \frac{1}{p!} A_{I_1 \cdots I_p} dx^{I_1} \wedge \cdots \wedge dx^{I_p}$ for differential forms. The role played by the 11d supergravity (SUGRA) in string theory has been a long-standing puzzle. A hint in this direction was that the S^1 circle dimensional reduction of 11d SUGRA to 10d gives exactly the nonchiral Type IIA SUGRA.

As all the 10d SUGRA theories are low-energy limits of the corresponding string theories, it was natural to ask for the existence and the properties of a quantum theory containing the 11d SUGRA as its low-energy limit. This (still unknown) theory was called M-theory and the study of its connection with string theories became a central goal for the string community in the last five years.

A natural conjecture was then logically put forward, namely that the Type IIA string in the strongly coupled regime is described by M-theory or, equivalently, that the M-theory compactified on S^1 is the 10d Type IIA superstring. For example, the bosonic fields of Type IIA SUGRA and that of the circle reduction of the bosonic 11d SUGRA contain both the graviton g_{AB} , antisymmetric tensor B_{AC} ($C_{11,AC}$ in 11d SUGRA), the dilaton Φ ($g_{11,11} \equiv (R_{11}M_{11})^2$ in 11d SUGRA), a one-form potential A_B ($g_{11,B}$) and a three-form potential C_{ABC} . By comparing the lagrangians of 11d SUGRA of Newton constant $M_{11} \sim k_{11}^{-2/9}$ compactified on a circle S^1 of radius R_{11} and of the Type IIA string of string scale M_{IIA} and string coupling λ_{IIA} , the following relations emerge⁴

$$R_{11}M_{11} = \lambda_{IIA}^{2/3} \quad , \quad g_{AB}^M = \lambda_{IIA}^{-2/3} g_{AB} \quad , \quad (62)$$

where g_{AB} and g_{AB}^M are the Type IIA string and the M-theory metric, respectively. These relations support the conjectured duality. In the weak-coupling regime of the Type IIA string ($\lambda_{IIA} \rightarrow 0$), $R_{11} \rightarrow 0$ and therefore the low-energy limit is indeed the 10d IIA SUGRA. On the other hand, in the strong coupling regime ($\lambda_{IIA} \rightarrow \infty$) a new-dimension decompactifies ($R_{11} \rightarrow \infty$), and the low-energy limit of the Type IIA string is described by 11d SUGRA. A second argument for the conjecture is motivated by trying to identify Kaluza-Klein modes of the 11d gravitational multiplet in string language. Using the mapping (62), the relation (8) can easily be proved. On the string side, these states are interpreted as D0 branes. A nontrivial check of the duality conjecture [59] is that a bound state of n D0 branes has a mass n times larger than the mass of a single D0 brane, in

⁴The second relation can equivalently be replaced by $M_{11} = \lambda_{IIA}^{-1/3} M_{IIA}$.

precise correspondence with the Kaluza-Klein spacing on the 11d supergravity side. More generally, it is known that the Type IIA string contains, in addition to the fundamental string states and to the solitonic NS fivebrane, D0,D2,D4,D6 and D8 BPS branes, coupling (electrically or magnetically) to the appropriate odd-rank antisymmetric tensors present in the massless spectrum of the theory. On the other hand, the 11d SUGRA contains M2 (membranes) and M5 (fivebranes) as classical solutions. A precise mapping between M-theory states compactified on the circle and Type IIA branes (with the exception of the Type IIA D8 brane) was achieved, and their corresponding tensions were found to be in agreement with the conjectured duality relations (62).

There is another possible compactification of M-theory to 10d, that preserves one-half of the original supersymmetry. Indeed, the 11d action (61) has the following symmetry

$$\begin{aligned} x_{11} &\rightarrow -x_{11} \ , \ \psi_I(-x_{11}) = \Gamma_{11}\psi_I(x_{11}) \ , \\ g_{AB}(-x_{11}) &= g_{AB}(x_{11}) \ , \ g_{11,A}(-x_{11}) = -g_{11,A}(x_{11}) \ , \\ C_{ABC}(-x_{11}) &= -C_{ABC}(x_{11}) \ , \ C_{11,AB}(-x_{11}) = C_{11,AB}(x_{11}) \ , \end{aligned} \quad (63)$$

where $A, B, C = 1 \cdots 10$ are ten-dimensional indices and $\Gamma_{11} = \Gamma_1 \cdots \Gamma_{10}$. We can then compactify on an *orbifold*, the interval S^1/Z_2 obtained by identifying opposite points, of coordinates x_{11} and $-x_{11}$, on the circle. The two fixed points of this operation, $x_{11} = 0$ and $x_{11} = \pi R_{11}$ play a peculiar role, as will be seen in a moment. From a 10d viewpoint, Γ_{11} acts as a chiral projector and selects one-half of the original gravitino, namely one (chiral) Majorana-Weyl spinor ψ_A with $\Gamma^A\psi_A$ projected out and another Majorana-Weyl spinor (of opposite chirality) ψ_{11} . The two spinors can be assembled into a 10d Majorana gravitino ψ_A containing 64 degrees of freedom. The full massless gravitational spectrum of M-theory on S^1/Z_2 includes also the 10d graviton g_{AB} , the dilaton ϕ contained in $g_{11,11} = (R_{11}M_{11})^2 = e^{4\phi/3}$ and an antisymmetric tensor field $C_{11,AB}$. The sum of the bosonic degrees of freedom adds up to 64, as expected by supersymmetry. This cannot be the end of the story, however. It is well-known that the massless 10d gravitino gives an anomaly

under the 10d diffeomorphisms, whereas the massive Kaluza-Klein modes are nonchiral and do not contribute to the anomaly. On the other hand, in a smooth 11d space there is no such anomaly. A natural possibility is that the anomaly draws its origin from the two ends of the interval and is equally distributed between the fixed points $x_{11} = 0$ and $x_{11} = \pi R_{11}$. A standard explicit computation then asks for 496 Majorana-Weyl fermions, 248 on each of the fixed points, to cancel it. These fermions come necessarily from super Yang-Mills vector multiplets and can be associated to the gauge group $E_8 \times E_8$, with one gauge factor per fixed point. The bosonic part of the Yang-Mills action is then

$$L_{SYM} = -\frac{1}{\lambda_1^2} \int_{x_{11}=0} d^{10}x \sqrt{g} \operatorname{tr} F_1^2 - \frac{1}{\lambda_2^2} \int_{x_{11}=\pi R_{11}} d^{10}x \sqrt{g} \operatorname{tr} F_2^2 , \quad (64)$$

where λ_i are the two Yang-Mills couplings. Similarly to the weakly-coupled heterotic string, supersymmetry invariance of the action ask for a modification of the Bianchi identity associated to the three-form. A closer look at the SUGRA-SYM Lagrangian requires that the modification to the Bianchi identity be concentrated on the fixed planes and read [27]

$$dG = \frac{k_{11}^2}{\sqrt{2}\lambda^2} \{ \delta(x_{11}) (\frac{1}{2} \operatorname{tr} R^2 - \operatorname{tr} F_1^2) + \delta(x_{11} - \pi R_{11}) (\frac{1}{2} \operatorname{tr} R^2 - \operatorname{tr} F_2^2) \} . \quad (65)$$

A consistent M-theory compactification is obtained by using

$$\int_{C_5} dG = 0 \quad , \quad \int_{C_4^i} (\operatorname{tr} F_i^2 - \frac{1}{2} \operatorname{tr} R^2) = m_i - \frac{1}{2} p_i , \quad (66)$$

for any closed 5-cycle C_5 and arbitrary 4-cycle C_4^i defined at the fixed points $x_{11}^i = 0, \pi R_{11}$, where in (66) m_i, p_i are integers. The two equations (66) define thus the embedding of the spin connection into the gauge group as one particular solution of the equation

$$m_1 + m_2 = \frac{1}{2} (p_1 + p_2) . \quad (67)$$

Gauge and gravitational anomaly cancellation issues was first discussed by Horava and Witten [27, 60], starting from a particular solution to the Bianchi identity (65). It was later realized [61, 62] that the original solution [27] is not unique and that a one-parameter class a solutions exist, parametrized by b in the following. A critical reanalysis of anomaly

cancellation appeared recently [63], which insists on a periodic global definition of various M-theory fields. In particular, [63] uses a periodic generalization of the $\epsilon(x_{11})$ function on the interval $-\pi R_{11} \leq x_{11} \leq \pi R_{11}$, whose definition and derivative are

$$\epsilon_1(x_{11}) = \text{sign}(x_{11}) - \frac{x_{11}}{\pi R_{11}} \quad , \quad d\epsilon_1 = [2\delta(x_{11}) - \frac{1}{\pi R_{11}}]dx_{11} \quad . \quad (68)$$

The above defined $\epsilon_1(x_{11})$ is indeed periodic and continuous at $x_{11} = \pi R_{11}$ and has a step-type discontinuity at $x_{11} = 0$. Similarly, another function discontinuous at $x_{11} = \pi R_{11}$ can be defined by $\epsilon_2(x_{11}) = \epsilon_1(x_{11} - \pi R_{11})$. With these definitions, the solution to the Bianchi identity (65) reads [63]

$$G = 6 \, dC + \frac{k_{11}^2}{\lambda^2} \left\{ (b-1) \sum_{i=1}^2 \delta_i \wedge Q_3^i + \frac{b}{2} \sum_{i=1}^2 \epsilon_i \hat{I}_4^i - \frac{b}{2\pi} dx_{11} \wedge \sum_{i=1}^2 Q_3^i \right\} \quad , \quad (69)$$

where we used the following definitions $\hat{I}_4^i = (1/2)\text{tr}R^2 - \text{tr}F_i^2$, $Q_3^i = (1/2)\omega_{3L} - \omega_{3Y}^i$ (ω_{3L} and ω_{3Y}^i are Lorentz and gauge Chern-Simons forms, respectively), $\delta_1 \equiv \delta(x_{11})dx^{11}$, etc. It is useful to remember that these definitions are such that $\hat{I}_4^i = dQ_3^i$. The parameter b can be fixed by a global argument [63]

$$\int_{C_5} dG = \int_{C_4(x_{11}^{(1)})} G - \int_{C_4(x_{11}^{(2)})} G \quad , \quad (70)$$

where the 5-cycle C_5 has the boundary $\partial C_5 = C_4(x_{11}^{(2)}) + C_4(x_{11}^{(1)})$, $-\pi R_{11} < x_{11}^{(1)} < 0$ and $0 < x_{11}^{(2)} < \pi R_{11}$. If the standard embedding condition $m_1 = p_1 = p_2$, $m_2 = 0$ is not satisfied, then an explicit evaluation of (70) using (65) and (69) forces upon $b = 1$. The gauge and gravitational anomalies are concentrated on the boundaries and are given by the standard 10d expressions. Surprisingly enough, the Green-Schwarz term taking care of their compensation is the 11d topological Chern-Simons term in (61), since C is not Yang-Mills and Lorentz invariant. In the gauge variation of the three-form C there is actually an additional arbitrariness [61]. Making for simplicity the gauge choice $\delta C_{ABC} = 0$, the cancellation between the 10d one-loop anomaly and the tree-level gauge variation of the Chern-Simons term fixes the relation between the Yang-Mills couplings and the 11d

gravitational coupling to be [63]

$$\frac{k_{11}^4}{\lambda^6} = \frac{12}{(4\pi)^5} . \quad (71)$$

Compactifications of M-theory can be defined deforming [28] around a space of the form $S^1/Z_2 \times X^6$, with X^6 a Calabi-Yau space of Hodge numbers $(h_{(1,1)}, h_{(2,1)})$, in a perturbative expansion in $k_{11}^{2/3}$. We denote in the following by $i, j = 1, 2, 3$ the complex Calabi-Yau indices and by μ, ν, ρ the 4d spacetime indices. The resulting 5d bulk theory contains as bosonic fields the gravitational multiplet, the universal hypermultiplet $(\det g_{i\bar{j}}, C_{\mu\nu\rho}, C_{ijk} \equiv \epsilon_{ijk}a)$, with a a complex scalar, $h_{(2,1)}$ additional hypermultiplets $(g_{ij}, C_{ij\bar{k}})$ and $h_{(1,1)} - 1$ vector multiplets $(C_{\mu i\bar{j}}, g_{i\bar{j}})$, with the determinant $\det g_{i\bar{j}}$ of the metric removed here and included in the universal hypermultiplet. The effect of the nontrivial Bianchi identity is to produce potential terms for moduli fields such that the 5d theory becomes a gauged SUGRA [33]. The spectrum on the two boundaries depends on the solution chosen for (67). For example, the standard embedding solution $m_1 = p_1 = p_2$ and $m_2 = 0$ gives a gauge group E_6 on one boundary with $h_{(1,1)}$ chiral multiplets in the fundamental representation **27** of E_6 and $h_{(2,1)}$ chiral multiplets in the **$\overline{27}$** , while the other boundary hosts a super Yang-Mills theory with gauge group E_8 . Nonstandard embeddings and nonperturbative vacua containing fivebranes were also considered [64].

Some orbifold compactifications of M-theory of the type $T^n/Z_2 \times X^{7-n}$ were also considered in the literature [65]. On the other hand, compactifications on particular compact spaces $S^1/Z_2 \times S^1 \times X^5$ can be simply related to Type I compactifications. In order to see this, it is enough to study the compactification to 9d. The compactification of M-theory on $S^1 \times S^1/Z_2$ (with radii R_{10} and R_{11} , respectively) admits two different interpretations [27]:

1. as M-theory on $S^1/Z_2 \times S^1$, that according to [27] describes the $E_8 \times E_8$ heterotic string of coupling $\lambda_{E_8} = (R_{11}M_{11})^{3/2}$, compactified on a circle S^1 of radius $R_{E_8} = R_{10}(R_{11}M_{11})^{1/2}$. In this case, a Wilson line must be added, and the theory is in a

vacuum with an unbroken $SO(16) \times SO(16)$ gauge group. By making a standard T-duality transformation $R_H = 1/R_{E_8} M_H^2$, $\lambda_H = \lambda_{E_8}/R_{E_8} M_H$, we can relate it to the $SO(32)$ heterotic string in the vacuum state with gauge group $SO(16) \times SO(16)$, of coupling $\lambda_H = R_{11}/R_{10}$ and radius $R_H = 1/(R_{10}(R_{11}M_{11})^{1/2})M_H^2$.

2. as M-theory on $S^1 \times S^1/Z_2$, that according to [45] describes the IIA theory of coupling $\lambda_{IIA} = (R_{10}M_{11})^{3/2}$, compactified further on the S^1/Z_2 orientifold of radius $R_{11}(R_{10}M_{11})^{1/2}$. The result is the Type-I' theory, T-dual (with respect to the eleventh coordinate) to the Type I theory (in its $SO(16) \times SO(16)$ vacuum), with coupling $\lambda_I = R_{10}/R_{11}$, compactified on a circle of radius $1/(R_{11}(R_{10}M_{11})^{1/2})M_I^2$. In the M-theory regime ($R_{11} \gg R_{10}$), the Type I and Type I' theories can both be weakly coupled, and can consequently be treated as perturbative strings.

It is interesting to notice that the above duality relations are in agreement with the $SO(32)$ heterotic-Type I duality conjecture $\lambda_H = 1/\lambda_I$, $R_H = R_I/\lambda_I^{1/2}$, which can therefore be regarded as a prediction in this framework.

A further check of these duality chains is found translating in Type I or heterotic language the masses of the BPS states of M-theory [27]. Consider first the Kaluza-Klein states of the supergravity multiplet on $T^2 = S^1/Z_2 \times S^1$, together with the wrapping modes of the M2 membrane around the torus. Their masses are

$$\mathcal{M}^2 = \frac{l^2}{R_{11}^2} + \frac{m^2}{R_{10}^2} + n^2 R_{10}^2 R_{11}^2 M_{11}^6 \quad , \quad (72)$$

where (l, m, n) is a triplet of integers labelling the corresponding charges. These masses must have a clear physical interpretation on the Type I and heterotic sides. In Type-I and Type-I' units, the masses of the states (72) are

$$\mathcal{M}_I^2 = l^2 R_I^2 M_I^4 + \frac{m^2 R_I^2 M_I^4}{\lambda_I^2} + \frac{n^2}{R_I^2} \quad , \quad \mathcal{M}_{I'}^2 = \frac{l^2}{R_{I'}^2} + \frac{m^2 M_{I'}^2}{\lambda_{I'}^2} + n^2 R_{I'}^2 M_{I'}^4 \quad . \quad (73)$$

In a similar fashion, in $E_8 \times E_8$ and $SO(32)$ heterotic units the states (72) have masses

$$\mathcal{M}_{E_8}^2 = \frac{l^2 M_H^2}{\lambda_{E_8}^2} + \frac{m^2}{R_{E_8}^2} + n^2 R_{E_8}^2 M_H^4 \quad , \quad \mathcal{M}_H^2 = l^2 \frac{R_H^2 M_H^4}{\lambda_H^2} + m^2 R_H^2 M_H^4 + \frac{n^2}{R_H^2} \quad . \quad (74)$$

Notice that KK modes l along the eleventh dimension are, according to (74), nonperturbative in heterotic units but are perturbative states in Type I and Type I' units (73). In particular, a Scherk-Schwarz type breaking $l \rightarrow l + \omega$, with some fractional number ω , describes nonperturbative heterotic physics [32],[34] but can be perturbatively described in Type I strings [19], as we will show in detail in Section 7.

There are also twisted M-theory states associated to the fixed points of S^1/Z_2 , that are charged under the gauge group. These include ordinary momentum excitations in the tenth direction and membrane wrappings in the full internal space, for which

$$\mathcal{M}^2 = \frac{\tilde{m}^2}{R_{10}^2} + \tilde{n}^2 R_{10}^2 R_{11}^2 M_{11}^6 \quad . \quad (75)$$

In type I and Type I' units, their masses become

$$\mathcal{M}_I^2 = \frac{\tilde{m}^2 R_I^2 M_I^4}{\lambda_I^2} + \frac{\tilde{n}^2}{R_I^2} \quad , \quad \mathcal{M}_{I'}^2 = \frac{\tilde{m}^2 M_I^2}{\lambda_{I'}^2} + \tilde{n}^2 R_{I'}^2 M_I^4 \quad , \quad (76)$$

and the wrapping modes are thus perturbative open string states. In $E_8 \times E_8$ and $SO(32)$ heterotic units, the masses of the charged states are

$$\mathcal{M}_{E_8}^2 = \frac{\tilde{m}^2}{R_{E_8}^2} + \tilde{n}^2 R_{E_8}^2 M_H^4 \quad , \quad \mathcal{M}_H^2 = \tilde{m}^2 R_H^2 M_H^4 + \frac{\tilde{n}^2}{R_H^2} \quad . \quad (77)$$

The perturbative states labeled by n and \tilde{n} have counterparts in the Type I theory that reflect the perturbative heterotic-Type I duality (see eqs. (73) and (76)). This is effective if the string coupling λ_I is small and R_I is large.

The supersymmetric S^1/Z_2 compactification is not the only possibility compatible with the Z_2 orbifold structure. Indeed, there is the possibility of a nontrivial, Scherk-Schwarz type 11d \rightarrow 10d compactification, obtained giving a nontrivial y_{11} dependence to the zero modes in the Kaluza-Klein expansion [34, 32]. This is consistent if the 11d theory has an appropriate discrete symmetry, that in this case is the fermion number. Then the 11d gravitino field $\Psi = (\Psi_1, \bar{\Psi}_2)^T$, where Ψ_1, Ψ_2 are the two Majorana-Weyl spinors, can have

the nontrivial KK decomposition

$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = U \begin{pmatrix} \sum_{m=0}^{\infty} \cos \frac{my_{11}}{R_{11}} \Psi_1^{(m)} \\ \sum_{m=1}^{\infty} \sin \frac{my_{11}}{R_{11}} \Psi_2^{(m)} \end{pmatrix}, \quad (78)$$

where $U \equiv \exp(My_{11})$ and M is an antisymmetric matrix. Compatibility of the truncation (78) with the orbifold symmetry Z_2 requires $\{Z_2, M\} = 0$, which fixes M to be the off-diagonal antisymmetric matrix [32] $M = i\omega\sigma_2 M_{11}$, where σ_2 is the Pauli matrix and $\omega = 1/2$ is fixed by the requirement that $U(y_{11} = 2\pi R_{11}) = -I$. The Scherk-Schwarz decomposition in this case reads explicitly

$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} \cos \frac{y_{11}}{2R_{11}} & \sin \frac{y_{11}}{2R_{11}} \\ -\sin \frac{y_{11}}{2R_{11}} & \cos \frac{y_{11}}{2R_{11}} \end{pmatrix} \begin{pmatrix} \sum_{m=0}^{\infty} \cos \frac{my_{11}}{R_{11}} \Psi_1^{(m)} \\ \sum_{m=1}^{\infty} \sin \frac{my_{11}}{R_{11}} \Psi_2^{(m)} \end{pmatrix}, \quad (79)$$

and indeed breaks supersymmetry in the eleventh dimension. Notice that the surviving gravitini on the two boundaries $y_{11} = 0$ and $y_{11} = \pi R_{11}$ have opposite chirality [66]. The same result holds for the supersymmetric spinor transformation parameter. Therefore, in order to compensate the gauge and the gravitational anomalies on the two boundaries we must introduce as usual the $E_8 \times E'_8$ gauge group, but the chiralities of the gauginos in the two gauge factors are different. Each boundary preserves one-half of the original 11d supersymmetry, but the configuration containing both of them breaks supersymmetry completely ⁵. In Section 7 we will present, by compactifying down to 9d, a Type I string description of this phenomenon [19] and in Section 10 a 4d compactified description at the field theory level. It will be shown in Section 7 that the chirality flip means that one boundary contains branes and the other boundary antibranes, mutually interacting. Interestingly, this field theoretic argument proves that in even spacetime dimensions, where we can define Weyl fermions, the breaking of supersymmetry by compactification in one direction Y perpendicular to the branes is consistent only if the zero mode Y -variation of bulk fermions gives precisely Weyl fermions at the position of the branes Y_i , while for arbitrary bulk positions Y the zero modes have no definite chirality.

⁵This argument is equivalent to the one recently presented in [67].

5. Type I supersymmetric compactifications to four-dimensions

A particularly simple way of reducing the number of supersymmetries and of producing fermion chirality is to compactify on orbifolds [4]. A d-dimensional orbifold O^d can be constructed starting with the d-dimensional euclidean space R^d or the d-dimensional torus T^d and identifying points as

$$O^d = R^d/S = T^d/P \quad , \quad (80)$$

where the *space group* S contains rotations θ and translations \mathbf{v} and the *point group* P is the discrete group of rotations obtained from the space group ignoring the translations. A typical element of S acts on coordinates as $X \rightarrow \theta X + \mathbf{v}$ and is usually denoted (θ, \mathbf{v}) . The subgroup of S formed by pure translations $(1, \mathbf{v})$ is called the *lattice* Γ of S . The identification of points of R^d under Γ defines the torus T^d . Points of T^d can then be further identified under P to form the orbifold O^d . This is clearly consistent only if P consists of rotations which are automorphisms of the lattice Γ .

In most of the following sections we will be interested in 4d $\mathcal{N} = 1$ orientifolds obtained by orbifolding the six real (three complex) internal coordinates by the twist $\theta = (e^{2i\pi v_1}, e^{2i\pi v_2}, e^{2i\pi v_3})$, where $\mathbf{v} \equiv (v_1, v_2, v_3)$ is called the twist vector and where for a Z_N orbifold $\theta^N = 1$. If $v_1 + v_2 + v_3 = 0$ with all $v_i \neq 0$, the orientifold has generically $\mathcal{N} = 1$ supersymmetry (the $\mathcal{N} = 2$ of the parent Type IIB model broken to half of it by the orientifold projection Ω) while if, for example, $v_3 = 0$ and $v_1 + v_2 = 0$, the corresponding orientifold has $\mathcal{N} = 2$ supersymmetry. The group structure of the orientifolds we use in the following⁶ is $(1, \Omega, \theta^k, \Omega\theta^k \equiv \Omega_k)$. The independent models were classified long time ago [4] and in 4d the $\mathcal{N} = 1$ orientifolds are $Z_3, Z_4, Z_6, Z'_6, Z_7, Z_8, Z'_8, Z_{12}, Z'_{12}$ and $Z_N \times Z_M$ for some integers N and M . All of them contain a set of 32 D9 branes. In addition, the ones containing Z_2 -type elements ($Z_4, Z_6, Z'_6, Z_8, Z_{12}, Z'_{12}$) have sets of 32 D5 branes, needed

⁶The group structure is however not unique, see for example [68].

here for the perturbative consistency of the compactified theory. The presence of the D5 branes can be understood as follows. The orientifold group element $\Omega\theta^{N/2}$ (and sometimes other elements, too) has fixed (hyper)planes called $O5_+$ planes, negatively charged under the (twisted) RR fields. By flux conservation, they ask for a corresponding set of D5 branes with opposite RR charge. The actual position of the D5 branes is not completely fixed. They can naturally sit at the orbifold fixed points or they can live “in the bulk” in sets of $2N$ branes in a Z_N orbifold. This brane displacement [14, 68, 20] can be understood as a Higgs phenomenon breaking the open string gauge group and the sets of $2N$ bulk branes can be regarded as one brane and its various images through the orbifold and orientifold operations.

The three new (in addition to the torus) Type I one-loop amplitudes for a Z_N orientifold can be written generically as

$$\begin{aligned}\mathcal{K} &= \frac{1}{2N} \sum_{k=0}^{N-1} \int_0^\infty \frac{dt}{t} \int \frac{d^4p}{(2\pi)^4} Str_{closed} \Omega \theta^k q^{\alpha'(p^B p_B + m^2)} \equiv \int \frac{dt}{t} (4\pi^2 \alpha' t)^{-2} K , \\ \mathcal{A} &= \frac{1}{2N} \sum_{i,j=1}^{32} \sum_{k=0}^{N-1} \int_0^\infty \frac{dt}{t} \int \frac{d^4p}{(2\pi)^4} Str_{(i,j)} \theta^k q^{\alpha'(p^B p_B + m^2)} \equiv \int \frac{dt}{t} (8\pi^2 \alpha' t)^{-2} A , \\ \mathcal{M} &= \frac{1}{2N} \sum_{i=1}^{32} \sum_{k=0}^{N-1} \int_0^\infty \frac{dt}{t} \int \frac{d^4p}{(2\pi)^4} Str_{(i,i)} \Omega \theta^k q^{\alpha'(p^B p_B + m^2)} \equiv \int \frac{dt}{t} (8\pi^2 \alpha' t)^{-2} M , \quad (81)\end{aligned}$$

where the modular parameters for the three one-loop surfaces are defined in (29),(32) and (36). Ω acts on the open string oscillators as $\Omega\alpha_m = \pm e^{i\pi m} \alpha_m$, with the upper plus sign for the NN open sector and the lower minus sign for the DD open sector. The supertrace takes into account, as usual, the different statistics of bosons and fermions $Str \sim \sum_{bos} - \sum_{ferm}$ and the 4d momentum integrals give rise to the factors $(8\pi^2 \alpha' t)^{-2}$ in \mathcal{A} and \mathcal{M} (and to the corresponding one in \mathcal{K}). The projection operator \mathcal{P} introduced in Section 3, eqs. (25), (28), (33) and (37) for an Z_N orbifold reads

$$\mathcal{P} = \frac{1}{N} (1 + \theta + \dots + \theta^{N-1}) , \quad (82)$$

and therefore projects into orbifold invariant states $\mathcal{P}|phys\rangle = |phys\rangle$. The untwisted

massless closed spectrum is found by first displaying the right (and left) massless states

<i>Sector</i>	<i>State</i>	θ^k	helicity
NS	$: \quad \Psi_{-1/2}^\mu 0\rangle$	1	± 1
NS	$: \quad \Psi_{-1/2}^{j,\pm} 0\rangle$	$e^{\pm 2\pi i k v_j}$	6×0
R	$: \quad s_0 s_1 s_2 s_3\rangle$	$e^{2\pi i k (s_1 v_1 + s_2 v_2 + s_3 v_3)}$	$4 \times (\pm \frac{1}{2})$

(83)

where $s_i = \pm 1/2$, $s_0 + s_1 + s_2 + s_3 = 0 \pmod{2}$ is the GSO projection in the R sector and $j = 1, 2, 3$ denote (complex) compact indices. The physical closed string spectrum is obtained taking left-right tensor products $|L\rangle \otimes |R\rangle$ invariant under the orbifold and orientifold involution. Typically the NS-NS spectrum of the orientifold is symmetrized by Ω , while the RR spectrum is antisymmetrized, but other choices are possible.

The action of a twist element θ^k in the open N and D sectors can be described by 32×32 matrices $\gamma_{\theta^k} \equiv \gamma^k = (\gamma_\theta)^k$ acting on the Chan-Paton degrees of freedom⁷ $\lambda^{(0)}$ for gauge bosons and $\lambda^{(i)}$ ($i = 1, 2, 3$) for matter scalars. Imposing that vertex operators for the corresponding physical states be invariant under the orbifold projection defines this action to be

$$\theta^k : \quad \lambda^{(0)} \rightarrow \gamma^k \lambda (\gamma^k)^{-1}, \quad \lambda^{(i)} \rightarrow e^{2\pi i k v_i} \gamma^k \lambda (\gamma^k)^{-1}. \quad (84)$$

Since $\theta^N = 1$, it follows from (84) that $\gamma^N = \pm 1$. For $\gamma^N = 1$ the gauge groups in the D9 and D5 brane sectors are subgroups of $SO(32)$, while for $\gamma^N = -1$ the D9, D5 gauge groups are subgroups of $U(16)$. The two choices correspond, in the notation of the previous section, to “real” charges n and to pairs of complex charges (m, \bar{m}) . The corresponding contribution to the one-loop annulus amplitudes (81) is multiplied by a Chan-Paton multiplicity $(Tr \gamma^k)^2$.

⁷In the case of $B_{\mu\nu}$ backgrounds [51] and other discrete backgrounds [52] there is a reduction of the rank of the gauge group and for models with branes and antibranes the rank of the matrix can be arbitrary, as we shall see later on.

Similarly, for every element $\Omega\theta^k \equiv \Omega_k$ there is an associated matrix acting on the CP indices γ_{Ω_k} as

$$\Omega_k : \lambda^{(0)} \rightarrow -\gamma_{\Omega_k} (\lambda^{(0)})^T (\gamma_{\Omega_k})^{-1}, \quad \lambda^{(i)} \rightarrow -\gamma_{\Omega_k} (\lambda^{(i)})^T (\gamma_{\Omega_k})^{-1}. \quad (85)$$

Since $\Omega^2 = 1$ it follows also that $\gamma_{\Omega} = \pm\gamma_{\Omega}^T$. The corresponding Möbius amplitudes are multiplied by the multiplicity factor $Tr(\gamma_{\Omega_k}^{-1}\gamma_{\Omega_k}^T)$. Without loss of generality the matrices γ^k can be chosen to be diagonal. The tadpole consistency conditions fix the Chan-Paton matrices γ analogously to (41), which in turn determine the gauge group and the charged matter content of the corresponding 4d orientifold. A generic supersymmetric model contains in the closed and the open spectrum states having a 10d origin, having a compactification lattice depending on all six compact coordinates, called the $\mathcal{N} = 4$ sector. There could also exist states having a 6d origin, with a compactification lattice depending on two compact coordinates, called $\mathcal{N} = 2$ sectors. Finally, there are states without any excitations in the compact coordinates, forming the $\mathcal{N} = 1$ sectors.

While the structure of the tadpole conditions cannot be described in full generality, some generic results should however be mentioned. In all cases, the tadpole conditions corresponding to untwisted forms are proportional to

$$\begin{aligned} D9 & : \quad \left\{ \frac{1}{32}(Tr\gamma_9^0)^2 - 2Tr(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^T) + 32 \right\} V_1 V_2 V_3, \\ D5 & : \quad \left\{ \frac{1}{32}(Tr\gamma_5^0)^2 - 2Tr(\gamma_{\Omega_{N/2},5}^{-1}\gamma_{\Omega_{N/2},5}^T) + 32 \right\} \frac{V_1}{V_2 V_3}, \end{aligned} \quad (86)$$

where V_1, V_2, V_3 are the volumes of the compact torii and we considered a D5 brane parallel to the first torus T^1 and orthogonal to T^2, T^3 . The solution to these equations is $\gamma_9^0 = \gamma_5^0 = I_{32}$, $\gamma_{\Omega,9} = \gamma_{\Omega,9}^T$ and $\gamma_{\Omega_{N/2},5} = \gamma_{\Omega_{N/2},5}^T$, asking therefore for one set of D9 branes and, for $N = \text{even}$, of one set of D5 branes. It can also be shown that one can choose conventions such that

$$Tr(\gamma_{\Omega_k,9}^{-1}\gamma_{\Omega_k,9}^T) = Tr(\gamma_9^{2k}) \quad , \quad Tr(\gamma_{\Omega_k,5}^{-1}\gamma_{\Omega_k,5}^T) = -Tr(\gamma_5^{2k}). \quad (87)$$

For Z_N orientifolds with N an odd integer, the twisted tadpole conditions can be easily worked out, too. Indeed, by using explicit expressions of the partition function on the three relevant one-loop surfaces [69], one finds the tadpole conditions

$$\sum_k \left\{ 32 \prod_{i=1}^3 \sin 2\pi k v_i + 2 \prod_{i=1}^3 \sin \pi k v_i (Tr \gamma_9^k)^2 - 16 \prod_{i=1}^3 \sin \pi k v_i (Tr \gamma_9^{2k}) \right\} = 0 . \quad (88)$$

For odd N summing over twisted sectors k or over twisted sectors $2k$ is however equivalent. We use this in order to rewrite all contributions in (88) in terms of $Tr \gamma_9^k$. We also define the number of fixed points $N_k = 64(\prod_{i=1}^3 \sin \pi k v_i)^2$ in an orbifold. Then in odd orbifolds $N_k = N_{2k}$, implying $64(\prod_{i=1}^3 \cos \pi k v_i)^2 = 1$. By combining these results, we can rewrite the solution of (88) in the form

$$Tr \gamma^{2k} = 32 \prod_{i=1}^3 \cos \pi k v_i . \quad (89)$$

Some of the models, $Z_4, Z_8, Z_{12}, Z'_{12}$ ($Z_2 \times Z_2$ models with discrete torsion) have additional tadpoles from the Klein bottle [69] proportional to $1/V_3$ ($V_1 V_2 / V_3$), which cannot be cancelled by adding sets of D5 branes. Surprisingly, these models seem therefore inconsistent, but it will be shown later on that, at least some of them allow consistent perturbative realisations with D9 branes and $D\bar{5}$ (anti)branes, with supersymmetry broken on the antibranes.

Let us exemplify these results referring to the first 4d Type I chiral model [15], the Z_3 orientifold with twist vector $\mathbf{v} = (1/3, 1/3, -2/3)$. The model has 32 D9 branes and the twisted tadpole condition (89) reads $Tr \gamma^{2k} = -4$, for $k = 1, 2$. The solution of (89) is $\gamma = (\omega I_{12}, \omega^2 I_{12}, I_8)$, with $\omega = \exp(2\pi i/3)$. The untwisted closed spectrum consists of the dilaton, the NS-NS scalar fields $g_{i\bar{j}}$, $i, \bar{j} = 1, 2, 3$ and the RR axions $B_{\mu\nu}$, $B_{i\bar{j}}$. The twisted closed spectrum consists of 27 linear multiplets, one per fixed point.

The annulus amplitude in (81) for the Z_3 orientifold can be written

$$\mathcal{A} = \mathcal{A}_{\mathcal{N}=4} - \frac{1}{6} \sum_{k=1}^2 \int_0^\infty \frac{dt}{t} \mathcal{A}^{(k)}(q) , \quad (90)$$

where $\mathcal{A}_{\mathcal{N}=4}$ is the contribution of the $\mathcal{N} = 4$ supersymmetric open spectrum, and $\mathcal{A}^{(k)}$ is the contribution of the $\gamma^k \equiv (\gamma)^k$ sectors given by

$$\mathcal{A}^{(k)} = \frac{1}{8\pi^4 t^2} \sum_{\alpha, \beta=0, 1/2} \eta_{\alpha, \beta} \frac{\vartheta[\frac{\alpha}{\beta}]}{\eta^3} \prod_{i=1}^3 (-2 \sin \pi k v_i) \frac{\vartheta[\frac{\alpha}{\beta + k v_i}]}{\vartheta[\frac{1/2}{1/2 + k v_i}]} (\text{tr} \gamma^k)^2, \quad (91)$$

by using the definitions (244) in the Appendix. The Möbius amplitude can be similarly written as in (90) by substituting $\mathcal{A} \rightarrow \mathcal{M}$, with

$$\mathcal{M}^{(k)} = -\frac{1}{8\pi^4 t^2} \sum_{\alpha, \beta=0, 1/2} \eta_{\alpha, \beta} \frac{\vartheta[\frac{\alpha}{\beta}]}{\eta^3} \prod_{i=1}^3 (-2 \sin \pi k v_i) \frac{\vartheta[\frac{\alpha}{\beta + k v_i}]}{\vartheta[\frac{1/2}{1/2 + k v_i}]} (\text{tr} \gamma^{2k}). \quad (92)$$

Because of supersymmetry, the amplitudes (91), (92) vanish identically using modular identities.

The gauge group and the massless spectrum can be exhibited after expressing the partition functions (91),(92) in terms of conformal characters [15]. To this end, the 10d $SO(8)$ Lorentz characters are decomposed with respect to the $SO(2) \times SU(3) \times U(1)$ subgroup, where the $SO(2)$ factor corresponds to the (light-cone) spacetime modes, so that the orbifold action is

$$\begin{aligned} V_8 - S_8 &= C_0 + C_- + C_+ \quad , \quad \theta (V_8 - S_8) = C_0 + \omega C_- + \omega^2 C_+ \quad , \\ \theta^2 (V_8 - S_8) &= C_0 + \omega^2 C_- + \omega C_+ \quad , \end{aligned} \quad (93)$$

where C_0 are modular functions describing in 4d a chiral multiplet and C_+, C_- are functions describing 3 chiral multiplets each⁸. Then the amplitudes (91) and (92) read

$$\begin{aligned} A &= \frac{(N + M + \bar{M})^2}{6} (C_0 + C_- + C_+) \sum_{m_i} P_{m_i}^{(6)} + \\ &\frac{(N + \omega M + \bar{\omega} \bar{M})^2}{6} (C_0 + \omega C_- + \bar{\omega} C_+) + \frac{(N + \bar{\omega} M + \omega \bar{M})^2}{6} (C_0 + \bar{\omega} C_- + \omega C_+) \quad , \\ M &= -\frac{(N + M + \bar{M})}{6} (C_0 + C_- + C_+) \sum_{m_i} P_{m_i}^{(6)} - \\ &\frac{(N + \omega M + \bar{\omega} \bar{M})}{6} (C_0 + \omega C_- + \bar{\omega} C_+) - \frac{(N + \bar{\omega} M + \omega \bar{M})}{6} (C_0 + \bar{\omega} C_- + \omega C_+) \quad , \end{aligned} \quad (94)$$

⁸The functions C_0, C_{\pm} are defined [15] starting from $SU(3)$ level-one characters, from the four $SO(2)$ characters and from the 12 characters of the $\mathcal{N} = 2$ superconformal model with central charge $c = 1$.

where N, M are Chan-Paton factors and $P_{m_i}^{(6)}$ is the momentum (Kaluza-Klein) compactification lattice. The massless spectrum reads from (94)

$$\begin{aligned} A_0 + M_0 = & [M\bar{M} + \frac{N(N-1)}{2}](C_0)_0 + [N\bar{M} + \frac{M(M-1)}{2}](C_-)_0 \\ & + [NM + \frac{\bar{M}(\bar{M}-1)}{2}](C_+)_0, \end{aligned} \quad (95)$$

where the subscript 0 denotes the massless part of the characters. The Chan-Paton factors are fixed by the tadpole conditions (86), (89)

$$N + M + \bar{M} = 32 \quad , \quad N - \frac{1}{2}(M + \bar{M}) = -4 \quad , \quad (96)$$

with the solution $M = 12, N = 8$. Therefore (95) describes an $\mathcal{N} = 1$ chiral model with gauge group $U(12) \times SO(8)$ and chiral multiplets in the representations $3(\mathbf{12}, \mathbf{8})_1 + 3(\overline{\mathbf{66}}, \mathbf{1})_{-2}$, where the subscripts denote the charges of the (anomalous) $U(1)_X$ factor contained in $U(12)$.

Alternatively, in the formalism of [14] the gauge group and the massless matter spectrum can be found from the equations

$$\begin{aligned} \lambda^{(0)} &= \gamma \lambda^{(0)} \gamma^{-1} \quad , \quad \lambda^{(0)} = -\gamma_\Omega (\lambda^{(0)})^T \gamma_\Omega^{-1} \quad , \\ \lambda^{(i)} &= e^{2\pi i v_i} \gamma \lambda^{(i)} \gamma^{-1} \quad , \quad \lambda^{(i)} = -\gamma_\Omega (\lambda^{(i)})^T \gamma_\Omega^{-1} \quad . \end{aligned} \quad (97)$$

The matrix $\gamma_\Omega = (I_{12} \otimes \sigma_1, I_8)$ (where σ_1 is the first, off-diagonal and symmetric Pauli matrix) interchanges the roots of γ with their complex conjugates. Solving (97) we find that the gauge fields $\lambda^{(0)}$ are described by a general 12×12 matrix and by an 8×8 antisymmetric matrix, giving indeed the gauge group $U(12) \times SO(8)$. Each of the matter fields $\lambda^{(i)}$, on the other hand, are described by two 12×8 matrices and by one 12×12 antisymmetric matrix, describing, as before, chiral multiplets in the representations $3(\mathbf{12}, \mathbf{8})_1 + 3(\overline{\mathbf{66}}, \mathbf{1})_{-2}$.

6. Effective action and quantum corrections in Type I strings

The effective field theory Lagrangian and the quantum corrections in Type I orbifold compactifications have some distinctive features compared to the corresponding heterotic compactifications, which will be briefly reviewed in this section. First of all, it is important to realize that some of the closed string (twisted and untwisted) axion-type fields are components of antisymmetric tensors from the RR sector. Together with the NS-NS scalars and the corresponding NS-R fermions, these are naturally described (in an $\mathcal{N} = 1$ language) by linear multiplets. On the other hand, in the heterotic string only the dilaton superfield was described by a linear multiplet, while all the other moduli fields fitted into chiral multiplets.

- Generalized Green-Schwarz mechanism

Let us start by defining the Type I compactification moduli, obtained by a straightforward reduction of the Lagrangian (10). By defining complex coordinates $i = 1, 2, 3$ and the associated components of the metric, $G_i^{\alpha\beta}$, $\alpha, \beta = 1, 2$ (with the dimension of a squared radius), the dilaton S and the geometric moduli T_i, U_i for the three complex planes are [69]

$$S = a^{RR} + i \frac{\sqrt{G_1 G_2 G_3} M_I^6}{\lambda_I}, \quad U_i = \frac{G_i^{12} + i \sqrt{G_i}}{G_i^{22}}, \quad T_i = b_i^{RR} + i \frac{\sqrt{G_i} M_I^2}{\lambda_I}, \quad (98)$$

where $G_i \equiv \det G_i^{\alpha\beta}$ and a^{RR}, b_i^{RR} are axionic fields from the RR sector. Our first goal here is to compute the tree-level and the one-loop threshold corrections to the gauge couplings of the Chan-Paton gauge groups. We expect here surprises compared to the heterotic models, where the tree-level gauge couplings are universal, $1/g_a^2 = \text{Re } f_a = k_a \text{Re } S$ and the numbers k_a denote the Kac-Moody levels. For example, in the Z_3 model described in the previous section, the abelian $U(1)_X$ factor is anomalous and the mixed $U(1)_X G_a^2$ anomalies $(C_{SU(12)}, C_{SO(8)}, C_{U(1)}) = (1/4\pi^2)(-18, 36, -432)$ are incompatible with the standard 4d version of the Green-Schwarz mechanism [1]. The solution to this puzzle was proposed in [17], in analogy with the generalized Green-Schwarz mechanism found in 6d by Sagnotti

[18]. It was conjectured in [17] that the gauge fields in Z_3 do couple at tree-level to a linear symmetric combination M of the 27 closed twisted moduli

$$f_a = S + s_a M . \quad (99)$$

Under a $U(1)_X$ gauge transformation with (superfield) parameter Λ , there are cubic gauge anomalies. The generalized Green-Schwarz mechanism requires a shift of the combination M of twisted moduli [73, 70]

$$V_X \rightarrow V_X + \frac{i}{2}(\Lambda - \bar{\Lambda}) , \quad M \rightarrow M + \frac{1}{2} \epsilon \Lambda , \quad (100)$$

such that the gauge-invariant combination appearing in the Kähler potential is $i(M - \bar{M}) - \epsilon V_X$. The mixed anomalies are cancelled provided the following condition holds

$$\frac{\epsilon}{4\pi^2} = \frac{C_{SU(12)}}{s_{SU(12)}} = \frac{C_{SO(8)}}{s_{SO(8)}} = \frac{C_{U(1)_X}}{s_{U(1)_X}} . \quad (101)$$

By supersymmetry arguments, one can also write the D-terms which encode the induced Fayet-Iliopoulos term

$$V_D = \frac{g_X^2}{2} \left(\sum_A X_A K_A \Phi^A + \epsilon \frac{\partial K}{\partial M} M_P^2 \right)^2 , \quad (102)$$

where Φ^A denotes the set of charged chiral fields of $U(1)_X$ charge X_A and $K_A = \partial K / \partial \Phi^A$. It was shown in [17] that actually the mixed anomalies C_a are proportional to $tr(Q_X \gamma) tr(Q_a^2 \gamma)$, where Q_X, Q_a are gauge group generators of $U(1)_X$ and of the gauge group factor G_a , respectively. By an explicit check they showed that indeed this proportionality is valid, and therefore the fields playing a role in cancelling gauge anomalies are the twisted (linear combination of) fields M . Surprisingly, the dilaton S plays no role in anomaly cancellation, since, as $tr Q_X = 0$, it does not mix with the gauge fields. The actual computation of the coefficients s_a and ϵ (and therefore the check of the overall normalisation in (101)) was performed in [70], coupling the theory to a background spacetime magnetic field B . In this case, the relevant information is encoded in the vacuum energy, that is expanded in powers of the magnetic field

$$\Lambda(B) = -\mathcal{T} - \frac{1}{2} \left(\mathcal{K} + \mathcal{A}(B) + \mathcal{M}(B) \right)$$

$$\equiv \Lambda_0 + \frac{1}{2} \left(\frac{B}{2\pi} \right)^2 \Lambda_2 + \frac{1}{24} \left(\frac{B}{2\pi} \right)^4 \Lambda_4 + \dots . \quad (103)$$

Computing the divergent piece of the vacuum energy quartic in the magnetic field it was found, for the slightly more general case of odd Z_N orientifolds, that

$$\Lambda_4 = -\frac{24\pi^4}{N} \sum_{k=1}^{N-1} (\text{tr} Q^2 \gamma^k)^2 \prod_{i=1}^3 |\sin \pi k v_i| \int dl , \quad (104)$$

where the terms $(\text{tr} Q^4 \gamma^k)$ cancel exactly between the annulus and the Möbius. The result (104) can be easily generalized to arbitrary orientifold vacua. The interpretation of this term of the type $(\text{tr} F^2)^2$ is that twisted NS-NS fields $m_k = \text{Im } M_k$ (the blowing-up modes of the orbifold) appear at tree-level in the gauge kinetic function of the gauge group and generate at one-loop (tree-level in the transverse, closed string picture) a tadpole. Notice that the closed-string propagator for a canonically normalized scalar of mass M_c^2 is

$$\Delta_{\text{closed}} = \frac{\pi}{2} \int_0^\infty dl e^{-\frac{\pi l}{2} (p^\mu p_\mu + M_c^2)} , \quad (105)$$

with l the modulus of the cylinder. The divergence of an on-shell propagator can thus be written formally as $\frac{\pi}{2} \int^\infty dl$. By using this in (104), one can identify the additional tree-level contribution to the gauge couplings. The full tree-level expression is finally

$$\frac{4\pi^2}{g_{a,0}^2} = \frac{1}{\ell} + \sum_{k=1}^{[\frac{N-1}{2}]} s_{ak} m_k \quad (106)$$

$$= \frac{1}{\ell} + \sum_{k=1}^{[\frac{N-1}{2}]} \frac{8\pi^2}{\sqrt{2\pi N}} (\text{tr} Q_a^2 \gamma^k) \prod_{i=1}^3 |\sin \pi k v_i|^{1/2} m_k , \quad (107)$$

where ℓ is the Hodge dual of the axion ReS in (98). Analogously, the coefficient ϵ in (101), (102) can be found from the mixing between the gauge fields and the twisted RR antisymmetric tensors, which can be computed introducing a background magnetic field B' for the abelian gauge factor $U(1)_X$. Indeed, the quadratic term has a UV divergent part

$$\frac{B'^2}{4N\pi^2} \sum_{k=1}^{N-1} \prod_{i=1}^3 |\sin \pi k v_i| (\text{tr} Q_X \gamma^k)^2 \int dl . \quad (108)$$

By using the (gauge-fixed) propagator

$$\Delta^{\mu\nu,\rho\sigma}(k^2) \equiv \langle C^{\mu\nu} C^{\rho\sigma} \rangle = (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \frac{i}{k^2} , \quad (109)$$

for the (RR) antisymmetric-tensor moduli $C^{\mu\nu}$, by factorization of (108) we find at the orbifold point $m_k = 0$ the coupling

$$- \frac{1}{2\sqrt{2N}\pi^3} \sum_{k=1}^{\lfloor \frac{N-1}{2} \rfloor} \prod_{i=1}^3 |\sin \pi k v_i|^{\frac{1}{2}} (-i \text{tr} Q_X \gamma^k) \epsilon_{\mu\nu\rho\sigma} C_{\mu\nu}^k F_X^{\rho\sigma} , \quad (110)$$

confirming therefore that the dilaton, which would correspond to the $k = 0$ (untwisted) contribution in (110) does not mix with the anomalous gauge field. The $U(1)_X$ gauge boson thus becomes massive, breaking spontaneously the symmetry, even for zero VEV's of the twisted fields m_k . However, the corresponding global symmetry $U(1)_X$ remains unbroken⁹, since the Fayet-Iliopoulos terms vanish in the orbifold limit $m_k = 0$ [72]. From (110) we find

$$\epsilon = \sqrt{\frac{2}{N\pi^3}} \sum_k \prod_{i=1}^3 |\sin \pi k v_i|^{\frac{1}{2}} (-i \text{tr} Q_X \gamma^k) . \quad (111)$$

The above discussion generalizes easily to other models, with additional anomalous $U(1)_\alpha$ ($\alpha = 1 \cdots N_X$) and corresponding linear combinations of twisted moduli fields M_k coupling to the gauge fields. In this case the gauge kinetic function becomes

$$f_a = S + \sum_k s_{ak} M_k , \quad (112)$$

and (100) generalizes to

$$V_\alpha \rightarrow V_\alpha + \frac{i}{2} (\Lambda_\alpha - \bar{\Lambda}_\alpha) , \quad M_k \rightarrow M_k + \frac{1}{2} \epsilon_{k\alpha} \Lambda_\alpha , \quad (113)$$

in an obvious notation. The cancellation of the gauge anomalies $\text{tr} X_\alpha Q_a^2$ described by the coefficients $C_{\alpha a}$ asks for the generalized Green-Schwarz conditions

$$C_{\alpha a} = \frac{1}{4\pi^2} \sum_k s_{ak} \epsilon_{k\alpha} , \quad (114)$$

⁹This can protect proton decay in phenomenological models with low-string scale.

valid for each α, a . The gauge-invariant field combination appearing in the Kähler potential is $i(M_k - \bar{M}_k) - \sum_\alpha \epsilon_{k\alpha} V_\alpha$ and generates, by supersymmetry, the D-terms

$$V_D = \sum_\alpha \frac{g_\alpha^2}{2} \left(\sum_A X_A^\alpha K_A \Phi^A + \sum_k \epsilon_{k\alpha} \frac{\partial K}{\partial M_k} M_P^2 \right)^2 . \quad (115)$$

A similar analysis for gravitational anomalies in orientifold models can be found in [71]. The Kähler potential for the twisted moduli M_k has not yet been computed in orientifolds. It is however known that, close to the orientifold point $m_k = 0$, it starts with a quadratic term $K = \sum_k M_k^\dagger M_k$. It is certainly important to work out the full Kähler potential for twisted moduli in the various known orbifold examples and to study the consequences of (115), especially for phenomenological problems like fermion masses and mixings and for supersymmetry breaking in models with anomalous $U(1)$ symmetries.

- Threshold corrections

The one-loop threshold corrections to gauge couplings can also be computed by the same method [74, 70] and are related to the quadratic term in (103). The general structure of the corrections is

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{4\pi^2}{g_a^2(\mu_0)} + \Lambda_{2,a} \equiv \frac{4\pi^2}{g_a^2(\mu_0)} + \int_{1/\mu_0^2}^{1/\mu^2} \frac{dt}{4t} \mathcal{B}_a(t) , \quad (116)$$

with the upper and lower limits corresponding, respectively, to the IR and the UV regions in the open channel. It is more convenient technically to implement the infrared cutoff with the help of a function $F_\mu(t)$. In the transverse channel, for example, one possible choice is

$$F_\mu(l) = 1 - e^{-l/\mu^2} , \quad (117)$$

with the same cutoff for the two relevant diagrams, the annulus and the Möbius. As explained in [74, 50, 75], the integral must converge in the UV if all the tadpoles have been canceled globally, and if the background field has no component along an anomalous $U(1)$ factor. The potential IR divergences, on the other hand, are due to massless charged

particles circulating in the loop, so that

$$\lim_{t \rightarrow \infty} \mathcal{B}_a(t) = b_a \quad (118)$$

is the β -function coefficient of the effective field theory at energies much lower than the first massive threshold.

The threshold corrections encoded in the function $\mathcal{B}_a(t)$ can be computed in a generic $\mathcal{N} = 1$ model containing D9 and D5 branes [70]. The various contributions can, in analogy with the case of heterotic models [76], be grouped into two parts. The first comes from the $\mathcal{N} = 1$ sectors, i.e. sectors in which the orbifold operation acts in a nontrivial way on all three complex planes. This sector gets contributions both from the compactification lattice and from the string oscillator states. The second comes from the $\mathcal{N} = 2$ sectors, i.e. sectors in which the orbifold operation leaves one compact torus fixed and rotates the two others. This sector gets contributions only from the compactification lattice, more precisely from the compact torus left fixed by the orbifold operation. The result can be understood [74, 75] noticing that the oscillator states are non-BPS and only BPS states can contribute to the threshold corrections from $\mathcal{N} = 2$ sectors. The same is true for $\mathcal{N} = 4$ sectors, which however give vanishing contributions. The corresponding contribution can be computed in a closed form and the result, obtained sending the UV cutoff to infinity, turns out to be

$$\begin{aligned} \Lambda_{2,a} &= \frac{1}{12} \sum_i b_{ai}^{(\mathcal{N}=2)} \int_0^\infty \frac{dt}{t} F_\mu(t) \sum_{(m_i^1, m_i^2)} \left[4 e^{-\frac{\pi t}{\sqrt{G_i} \text{Im} U_i} |m_i^1 + U_i m_i^2|^2} - e^{-\frac{\pi t}{\sqrt{G_i} \text{Im} U_i} |m_i^1 + U_i m_i^2|^2} \right] \\ &= \frac{1}{3\pi} \sqrt{G_i} \sum_i b_{ai}^{(\mathcal{N}=2)} \int_0^\infty dl (1 - e^{-\frac{l}{\mu^2}}) \sum_{(n_i^1, n_i^2)} \left[e^{-\frac{\sqrt{G_i}}{\pi \text{Im} U_i} |n_i^2 + U_i n_i^1|^2 l} - e^{-\frac{\sqrt{G_i}}{\pi \text{Im} U_i} |n_i^2 + U_i n_i^1|^2 l} \right], \end{aligned} \quad (119)$$

where $b_{ai}^{(\mathcal{N}=2)}$ is the effective theory beta function coefficient of the corresponding $\mathcal{N} = 2$ sector¹⁰ and m_i^1, m_i^2 are Kaluza-Klein momenta of the compact torus T^i . By explicitly

¹⁰Notice that our definition of $b_{ai}^{(\mathcal{N}=2)}$ differs from the definition of ref. [76]. Our definition represents the contribution of the i th $\mathcal{N} = 2$ sector to the total beta function, and therefore equals $b_{ai}^{(\mathcal{N}=2)}/ind$ in their notation.

computing (119), we find the result

$$\begin{aligned} \Lambda_{2,a} = & -\frac{1}{4} \sum_i b_{ai}^{(\mathcal{N}=2)} \ln(\sqrt{G_i} |\eta(U_i)|^4 \text{Im} U_i \mu^2) = \\ & -\frac{1}{4} \sum_i b_{ai}^{(\mathcal{N}=2)} \ln \left[\left(\frac{\text{Im} S}{\text{Im} T_j} \frac{\text{Im} T_i}{\text{Im} T_k} \right)^{1/2} |\eta(U_i)|^4 \text{Im} U_i \frac{\mu^2}{M_I^2} \right] , \end{aligned} \quad (120)$$

with $j \neq k \neq i$.

The corrections (120) are similar to the heterotic ones [76] in the $\text{Im} T_i \rightarrow \infty$ limit, taking into account that on the heterotic side the complex structure moduli have the same definition (98), while

$$S = a + i \frac{\sqrt{G_1 G_2 G_3} M_H^6}{\lambda_H^2} , \quad T_i = b_i + i \sqrt{G_i} M_H^2 . \quad (121)$$

Taking the infrared limit of the threshold function $\mathcal{B}_a(t)$, by using (118) one can compute the beta function of the effective field theory in a generic $Z_N \mathcal{N} = 1$ orientifold. The result is

$$\begin{aligned} b_a = & \frac{4}{N} \sum_{k \neq N/2} [(\text{tr} Q_a^2 \gamma_9^k)(\text{tr} \gamma_9^k) - 2(\text{tr} Q_a^2 \gamma_9^{2k})] \left(\prod_{i=1}^3 \sin \pi k v_i \right) \sum_{j=1}^3 \frac{\cos \pi k v_j}{\sin \pi k v_j} \\ & + \frac{1}{N} \sum_{i, k \neq N/2} (\text{tr} Q_a^2 \gamma_9^k)(\text{tr} \gamma_{5_i}^k) \cos \pi k v_i + \frac{24}{N} \text{tr} Q_a^2 , \end{aligned} \quad (122)$$

where the last contribution in the right-hand side of (122) comes from the D9-D5 part of the cylinder $\mathcal{A}_{95}^{(0)}$ and from the Möbius with the insertion of a $\theta^{N/2}$ twist $\mathcal{M}_{99}^{(N/2)}$. The second line in (122) exists only for even N orientifolds. It can be checked case by case that (122) indeed agrees with the field-theoretical definition of the beta-function

$$b_a = -3T_a(G) + \sum_r T_a(r) , \quad (123)$$

where $T_a(r)$ denotes, as usual, the Dynkin index of the representation r .

The results presented above from [70] were derived for the D9 branes. They apply however, with minimal modifications, also to D5 branes in the appropriate models. For

example, for a D5 brane parallel to the third complex plane, the tree-level gauge couplings, analogous to (106), read

$$\frac{4\pi^2}{g_{a,0}^2} = \text{Im}T_3 + \sum_{k=1}^{[\frac{N-1}{2}]} s'_{ak} m_k . \quad (124)$$

If the D5 brane is stuck to a fixed plane of the orbifold, the coefficients s'_{ak} are different from zero only for twisted fields living at that particular fixed point (hyperplane). If the D5 brane is moved to the bulk, the coefficients s'_{ak} are all vanishing, since the corresponding gauge fields cannot couple to the twisted fields that are confined to the fixed points.

Some more phenomenological aspects of 4d orientifolds can be found, for example, in [77].

7. Type I string mechanisms for breaking supersymmetry.

Without D-branes, in heterotic and Type II models the only known perturbative mechanism to spontaneously break supersymmetry¹¹ in superstrings [78] is the string generalization of the Scherk-Schwarz mechanism [79]. In this case, there are tree-level gaugino masses $m_{1/2} = \omega/R$, where R is the compact radius used for the breaking and ω a parameter that is arbitrary in field theory but quantized in string theory. The reason for this is that the gauge fields live in the full (bulk) 10d space and directly feel [80] supersymmetry breaking. Phenomenological reasons ask therefore for radii of the TeV size, a rather unnatural possibility in heterotic models [81], since it asks for a string coupling of the order of 10^{32} .

On the other hand, the presence of D-branes in Type I models, with gauge fields and matter confined on them, offers new possibilities for breaking supersymmetry compared to the heterotic constructions. They can generically be classified into three classes:

- (i) Breaking by compactification.

¹¹Supersymmetry is also broken by orbifolding the internal space. However, the resulting breaking is not soft, in the sense that there is typically no trace of the original supersymmetry in the resulting spectrum.

Here there are two subclasses. In the first, the D brane under consideration is parallel to the direction of breaking and the massless D brane spectrum feels at tree-level supersymmetry breaking. This situation was called “Scherk-Schwarz” breaking in [19], since it is the analog of the heterotic constructions [78] and the spectrum is a discrete deformation of a supersymmetric model. The corresponding spectra have heterotic duals. In the second class, the D brane under consideration is perpendicular to the direction of the breaking and the massless D brane spectrum is supersymmetric at tree-level. This was called “M-theory breaking” in [19] (also called “Brane Supersymmetry” in [82], which proposed to extend the phenomenon to the whole massive spectrum, a situation then realized in [83]), since it describes in particular supersymmetry breaking in M-theory along the eleventh dimension [32, 34], as shown in Section 4. These models ask also for the presence of antibranes (and antiorientifold planes) in the spectrum, interacting with the branes. Supersymmetry breaking is transmitted by radiative corrections from the brane massive states or from the gravitational sector to the massless modes.

All RR and NS-NS tadpoles can be set to zero in both subclasses of these models and we shall confine our attention to this choice. Moreover, in these models the closed (gravitational) sector has a softly broken supersymmetry.

(ii) Models containing brane-antibrane systems: Brane supersymmetry breaking.

In these constructions [23, 24, 25], the closed (bulk) sector is exactly supersymmetric to lowest order. We can also distinguish here between two subclasses of models. In the first subclass, tadpole conditions, and therefore the consistency of the theory, require the introduction of antibranes in the system. The closed sector is supersymmetric but is different from the standard supersymmetric one. These models contain D9-D $\bar{5}$ tachyon-free brane configurations. In the second subclass, the closed sector is the standard supersymmetric one. The RR tadpole conditions ask therefore for a minimal number of D-branes and the whole spectrum can thus be supersymmetric. However, one can consistently intro-

duce additional brane-antibrane pairs of the same type that break supersymmetry. These configurations interact and are tachyonic, but if the branes and the antibranes are suitable separated, the tachyons can be lifted in mass.

(iii) Breaking by internal magnetic fields

Internal background magnetic fields H_i in a compact torus T^i (of radii $R_1^{(i)}, R_2^{(i)}$) can couple to the open string endpoints [21], carrying charges $q_L^{(i)}, q_R^{(i)}$ under H_i . Particles of different spin couple differently to the magnetic field and acquire different masses, breaking supersymmetry [22]. Defining $\pi\epsilon_i = \arctan(\pi q_L^{(i)} H_i) + \arctan(\pi q_R^{(i)} H_i)$, the mass splittings of all string states can be summarized by the formula

$$\delta m^2 = (2n + 1)|\epsilon_i| + 2\Sigma_i \epsilon_i, \quad (125)$$

where n are the Landau levels of the charged particles in the magnetic field and Σ_i are internal helicities. Possible values of the magnetic fields satisfy a Dirac quantization condition $H_i \sim k/(R_1^{(i)} R_2^{(i)})$. For weak fields, $\epsilon_i \simeq (q_L^{(i)} + q_R^{(i)}) H_i$ and the resulting mass splittings are inversely proportional to the area of the magnetized torus $m_{SUSY}^2 \sim k/(R_1^{(i)} R_2^{(i)})$ [22]. The spectrum generically contains charged tachyons coming from scalars having internal helicities $\Sigma_i = -1$ ($\Sigma_i = 1$) for positive (negative) magnetic field, which can however be avoided in special models. The mechanism can easily accomodate several magnetic fields pointing out in several compact torii and can also be implemented in orbifold models.

Models of type (ii) and (iii) are characterized by the fact that all RR tadpoles cancel, while some NS-NS tadpoles are left uncanceled. As discussed in Section 3, the proper interpretation of the NS-NS tadpoles is that scalar potentials are generated for appropriate NS-NS moduli fields.

We now turn to a more detailed presentation of the mechanisms (i) and (ii). For simplicity of notation, throughout this section *we leave implicit the contribution of transverse bossons*, $1/\eta^8$ in the 9d and 10d models and $1/\eta^4$ in the 6d model discussed in the third paragraph.

- Breaking by compactification I: direction parallel to the brane (Scherk-Schwarz breaking)

These models are constructed performing a Scherk-Schwarz deformation in the closed sector on the Kaluza-Klein momentum states $m \rightarrow m + \omega$. The brane under consideration is parallel to the breaking direction, i.e. it has associated momentum (KK) modes which feel a similar breaking. All these models contain the geometrical objects present in supersymmetric models, in particular 32 D9 branes and 32 O9₊ planes, and are discrete deformations of supersymmetric models.

The simplest such example is provided by a 9d model which, in the closed sector, can be described as a Type OB/g orbifold, the orbifold operation being $g = -(-1)^{G_L}(-1)^n$, where G_L is the (left) world-sheet fermion number. Alternatively, after a redefinition of the radius $R \rightarrow 2R$, the model can be described as the orbifold $IIB/g = (-1)^F I$, where $F = F_L + F_R$ is the spacetime fermion number and I is the shift $I : X_9 \rightarrow X_9 + \pi R$, acting on the states as $(-1)^m$. The closed spectrum is tachyon free for $R \geq M_I^{-1}$ and supersymmetry is restored in the $R \rightarrow \infty$ limit ¹²

The relevant amplitudes to consider, in the notations of (81), are

$$\begin{aligned} K_1 &= \frac{1}{2} (V_8 - S_8) \sum_m P_m , \\ A_1 &= \frac{n_1^2 + n_2^2}{2} \sum_m (V_8 P_m - S_8 P_{m+1/2}) + n_1 n_2 \sum_m (V_8 P_{m+1/2} - S_8 P_m) , \\ M_1 &= -\frac{n_1 + n_2}{2} \sum_m (\hat{V}_8 P_m - \hat{S}_8 P_{m+1/2}) , \end{aligned} \tag{126}$$

where, as usual, n_1, n_2 denote Chan-Paton charges and $P_m(P_{m+1/2})$ denote integer (half-integer) momentum states. The spectrum corresponds to a family of gauge groups $SO(n_1) \times SO(n_2)$, with $n_1 + n_2 = 32$ fixed by the tadpole conditions. For integer momentum levels,

¹²Some of the models in this paragraph were studied also in [84].

the spectrum consists of vectors¹³ in the representations $(\mathbf{n}_1(\mathbf{n}_1-1)/2, \mathbf{1}) + (\mathbf{1}, \mathbf{n}_2(\mathbf{n}_2-1)/2)$ and fermions in the representation $(\mathbf{n}_1, \mathbf{n}_2)$. On the other hand, for half-integer levels, the spectrum consists of fermions in the $(\mathbf{n}_1(\mathbf{n}_1-1)/2, \mathbf{1}) + (\mathbf{1}, \mathbf{n}_2(\mathbf{n}_2-1)/2)$ and vectors in the $(\mathbf{n}_1, \mathbf{n}_2)$.

This model can easily be understood as a discrete deformation by the fermion number $(-1)^F$ of the supersymmetric model described by

$$\begin{aligned} K_1 &= \frac{1}{2}(V_8 - S_8) \sum_m P_m , \\ A_1 &= (V_8 - S_8) \sum_m \left(\frac{n_1^2 + n_2^2}{2} P_m + n_1 n_2 P_{m+1/2} \right) , \\ M_1 &= -\frac{n_1 + n_2}{2} (V_8 - S_8) \sum_m P_m , \end{aligned} \tag{127}$$

obtained breaking the compactified $SO(32)$ Type I model with the Wilson line $W = (I_{n_1}, -I_{n_2})$. The fact that (126) is a discrete deformation of (127) reflects the spontaneous character of the breaking, which disappears in the decompactification limit $R \rightarrow \infty$. Moreover, in this model a scalar potential is induced that for large radius behaves as $1/R^9$, and thus dynamically tends to decompactify the theory to 10d and to restore supersymmetry.

A large class of models can be constructed compactifying on orbifolds [19, 20], with different patterns of supersymmetry breaking: $\mathcal{N} = 4 \rightarrow \mathcal{N} = 0$, $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$, $\mathcal{N} = 4 \rightarrow \mathcal{N} = 1$, $\mathcal{N} = 2 \rightarrow \mathcal{N} = 0$ and $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$.

- Breaking by compactification II: direction orthogonal to the brane (M-theory breaking or Brane Supersymmetry)

The starting point in constructing these models is a shift $n \rightarrow n + \omega$ in the winding modes of the closed sector of the parent Type IIB superstring. The brane under consideration is perpendicular to the direction of the breaking. This is easy to visualize in the T-dual picture, where windings shifts become standard momentum shifts, but the compact

¹³In 9d the 10d vector actually comprises a vector and a scalar.

direction becomes perpendicular to the brane. A simple prototype is again provided by a 9d example, that in the parent IIB theory is simply obtained by interchanging the KK momenta with the windings in the (Scherk-Schwarz) breaking. The open sector, on the other hand, is completely different, due to the momentum/winding asymmetry in the open sector resulting from the standard Ω projection.

The relevant amplitudes are in this case

$$\begin{aligned} K_2 &= \frac{1}{2} (V_8 - S_8) \sum_m P_{2m} + \frac{1}{2} (O_8 - C_8) \sum_m P_{2m+1} , \\ A_2 &= \frac{N_1^2 + N_2^2}{2} (V_8 - S_8) \sum_m P_m + N_1 N_2 (O_8 - C_8) \sum_m P_{m+1/2} , \\ M_2 &= -\frac{N_1 + N_2}{2} \hat{V}_8 \sum_m P_m + \frac{N_1 + N_2}{2} \hat{S}_8 \sum_m (-1)^m P_m \end{aligned} \quad (128)$$

and the tadpole conditions are $N_1 = N_2 = 16$, and are satisfied also in the $R \rightarrow 0$ limit.

Notice that the massless open spectrum is supersymmetric, since

$$A_2 + M_2 = \frac{N_1^2 + N_2^2}{2} (V_8 - S_8) - \frac{N_1 + N_2}{2} (\hat{V}_8 - \hat{S}_8) + \text{massive} . \quad (129)$$

The open spectrum is actually supersymmetric for all even momenta and describes a vector and a spinor in the adjoint of $SO(16) \times SO(16)$. On the other hand, for odd momenta the vector is again in the adjoint, while the spinor is in the symmetric representations $(\mathbf{135}, \mathbf{1}) + 2(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{135})$. Finally, there are scalars and spinors in the $(\mathbf{16}, \mathbf{16})$ representation with half-integer momenta.

The duality arguments of Section 4 associate the closed sector of this Type-I model, after a T-duality, to a Scherk-Schwarz deformation affecting the momenta of the Type-IIA string. In the corresponding Type-I' representation, however, the open strings end on D8-branes perpendicular to the direction responsible for the breaking of supersymmetry. Therefore, as we have just seen, all open string modes with even windings, and in particular the massless ones, are unaffected. The soft nature of this breaking is less evident than in the previous example, but the very soft nature of the radiative corrections was shown in

[19] and is related to the local tadpole cancellation properties of the model. Resorting again to the duality arguments of Section 4, it is clear that this breaking corresponds to a non-perturbative phenomenon on the heterotic side and realizes the Scherk-Schwarz deformation along the eleventh coordinate of M theory. An additional nice argument concerning the relation to M-theory is that, in addition to global tadpole conditions, the model satisfies local tadpole conditions in the breaking direction and therefore has a consistent $R \rightarrow 0$ limit, equivalent in M-theory language to $R_{11} \rightarrow \infty$.

It is interesting to notice from (128) that the model actually contains 16 D9 branes and 16 $\text{D}\bar{9}$ antibranes. In the T-dual picture ($R_\perp = 1/RM_I^2$) the 16 D8 branes are at the origin $y = 0$ and the 16 $\text{D}\bar{8}$ branes are at the orientifold fixed point $y = \pi R_\perp$. To be more precise, there are 16 D8 branes and 16 $\text{O}8_+$ planes at $y = 0$ and 16 $\text{D}\bar{8}$ antibranes and 16 $\text{O}\bar{8}_+$ antiplanes at $y = \pi R_\perp$, such that supersymmetry is still preserved in the vicinity of each fixed points $y = 0, \pi R_\perp$ where local tadpole conditions are satisfied as well. In order to substantiate this picture and to better define the notion of antibranes and antiorientifold planes, let us describe in more detail the effective lagrangian describing the interactions of SUGRA fields with branes/orientifold planes in this model. This can be easily done writing the amplitudes (128) in the tree-level (closed) channel

$$\begin{aligned}\tilde{K}_2 &= \frac{2^5}{2} (V_8 \sum_n W_{2n} - S_8 \sum_n W_{2n+1}) , \\ \tilde{A}_2 &= \frac{1}{2^6} \sum_n \{ [N_1 + (-1)^n N_2]^2 V_8 - [N_1 - (-1)^n N_2]^2 S_8 \} W_n , \\ \tilde{M}_2 &= -(N_1 + N_2) (\hat{V}_8 \sum_n W_{2n} - \hat{S}_8 \sum_n W_{2n+1}) .\end{aligned}\tag{130}$$

The effective lagrangian can be found by writing the vacuum energy (without the torus contribution)

$$\tilde{K}_2 + \tilde{A}_2 + \tilde{M}_2 = \frac{1}{64} \{ [N_1 - 16 + (-1)^n (N_2 - 16)]^2 V_8 - [N_1 - 16 - (-1)^n (N_2 - 16)]^2 S_8 \} \sum_n W_n .\tag{131}$$

By factorization the effective bulk-D8/O8 action is easily found and reads

$$S = \int d^{10}x \{ \sqrt{G} \mathcal{L}_{SUGRA} - (N_1 - 16) T_8 (\sqrt{G} e^{-\Phi} + A_9) \delta(y) - (N_2 - 16) T_8 (\sqrt{G} e^{-\Phi} - A_9) \delta(y - \pi R) \} , \quad (132)$$

where

$$\mathcal{L}_{SUGRA} = \frac{1}{2k_{10}^2} \{ e^{-2\Phi} [R + 4(\partial\Phi)^2] - \frac{1}{2 \times 10!} F_{10}^2 \} \quad (133)$$

is the bosonic Type I supergravity action. Notice that the corresponding fermionic fields are massive, due to the breaking of supersymmetry by compactification.

In (132)- (133), A_9 is the RR 9-form coupling to the D8 ($D\bar{8}$) and $O8_+$ ($O\bar{8}_+$) systems and F_{10} is its corresponding field strength, k_{10} defines the 10d Planck mass, T_8 is the D8 brane tension and G is the 10d metric. The change in sign in the RR part of the last term in (132) confirms precisely that the model contains 16 D8/O8 at $y = 0$ and 16 $D\bar{8}/O\bar{8}$ at $y = \pi R$.

Branes and antibranes attract each other and for sufficiently small (large) distances R_\perp (R), a tachyon appears [87], as is easily seen also in (128). The configuration is therefore in principle unstable, although a more detailed analysis of the potential for the radius is needed in order to settle this question. The full vacuum energy for large R_\perp can be estimated to be (after a rescaling $l \rightarrow l/R^2$) [19]

$$\Lambda \sim -\frac{1}{R_\perp^9} - M_I^9 R_\perp \int_0^\infty dl e^{-\pi R_\perp^2 M_I^2 l} \sum_n (-1)^n e^{\frac{-\pi l n^2}{4}} \simeq -\frac{1}{R_\perp^9} - \frac{\pi}{8} M_I^9 e^{-2\pi R_\perp M_I} , \quad (134)$$

where numerical coefficients were set to one in (134). In (134), the first term is the one-loop torus contribution piece proportional to $1/R_\perp^9$ and the second attractive term comes from (anti)branes-orientifold contributions proportional to $\exp(-R_\perp M_I)$, where the exponential suppression appears due to the local tadpole property of the model. There the induced potential for the radius does not seem to have a minimum. However, this conclusion cannot be reliably drawn from the approximate expression (134), and new effects can certainly appear, as for instance tachyon condensation, which could render the configuration stable.

To the best of our knowledge, this is the first construction of a perturbative Type I model containing simultaneously branes and antibranes¹⁴. Other models with different patterns of supersymmetry breaking can be constructed compactifying on orbifolds [19, 20] and some duality relations with heterotic constructions were investigated in [85].

- Brane Supersymmetry Breaking I: non-BPS stable configurations

As already mentioned, the main idea of Brane Supersymmetry Breaking is to put together branes and antibranes. In general, such systems contain (open string) tachyons stretched between branes and antibranes, that reflect the attractive force between them. However, in some models the consistency of the theory (the RR tadpole conditions) asks for D9-D5 stable, tachyon-free non-BPS systems. The simplest model of this type we are aware of is the T^4/Z_2 orbifold model with orientifold projection in the twisted sector with a flipped sign [23, 24].

Following [10], let us introduce the convenient combinations of $SO(4)$ characters

$$\begin{aligned} Q_o &= V_4 O_4 - C_4 C_4, & Q_v &= O_4 V_4 - S_4 S_4, \\ Q_s &= O_4 C_4 - S_4 O_4, & Q_c &= V_4 S_4 - C_4 V_4, \end{aligned} \quad (135)$$

which describe in 6d at massless level the propagation of a vector multiplet (Q_o), of a hypermultiplet (Q_v), of half of a hypermultiplet (Q_s) and where Q_c contains only massive particles. With these notations, there are two consistent inequivalent choices for the Klein bottle, described by

$$K_3 = \frac{1}{4} \left\{ (Q_o + Q_v) \left(\sum_{m_i} P_{m_i}^{(4)} + \sum_{n_i} W_{n_i}^{(4)} \right) + 2\epsilon \times 16(Q_s + Q_c) \left(\frac{\eta}{\theta_4} \right)^2 \right\}, \quad (136)$$

where $P^{(4)}$ ($W^{(4)}$) denotes the momentum (winding) lattice sum and $\epsilon = \pm 1$. For both choices of ϵ , the closed string spectrum has $(1, 0)$ supersymmetry, but the two resulting

¹⁴Notice, however, that models with branes and antibranes were constructed previously in orientifolds of Type O models, [10, 43].

projections are quite different. The usual choice ($\epsilon = 1$) [10, 14] leaves 1 gravitational multiplet, 1 tensor multiplet and 20 hypermultiplets, while $\epsilon = -1$ leaves 1 gravitational multiplet, 17 tensor multiplets and 4 hypermultiplets. The case $\epsilon = -1$ is our primary focus here, since in this case the flipped Ω action in the closed twisted sector is equivalent with the replacement $O5_+ \rightarrow O5_-$. However, $O5_-$ RR charge is positive and the RR tadpole conditions require for consistency the presence of 32 $D\bar{5}$ (anti)branes in the spectrum.

The annulus amplitude is simpler to understand in the transverse channel

$$\begin{aligned} \tilde{A}_3 = & \frac{2^{-5}}{4} \left\{ (Q_o + Q_v) \left(N^2 v \sum_{n_i} W_{n_i}^{(4)} + \frac{D^2 \sum_{m_i} P_{m_i}^{(4)}}{v} \right) + 2ND(Q'_o - Q'_v) \left(\frac{2\eta}{\theta_2} \right)^2 \right. \\ & \left. + 16(Q_s + Q_c) \left(R_N^2 + R_D^2 \right) \left(\frac{\eta}{\theta_2} \right)^2 + 8R_N R_D (V_4 S_4 - O_4 C_4 - S_4 O_4 + C_4 V_4) \left(\frac{\eta}{\theta_3} \right)^2 \right\}, \end{aligned} \quad (137)$$

where we introduced the Chan-Paton multiplicities N, R_N, D, R_D and the primed characters [19] are related by a chirality change $S_4 \leftrightarrow C_4$ to the unprimed ones defined in eq. (135). For the spacetime part, this simply means a change of fermion chirality, as shown in more detail below. Notice that (137) is identical in structure to the corresponding amplitude for the supersymmetric T^4/Z_2 Type I orbifold [10], [14], except that in the $D9-D\bar{5}$ sector the signs of the RR terms are reversed, in order to correctly take into account the (negative) charge of $D\bar{5}$ (anti)branes. The direct-channel annulus is obtained by an S-transformation, and reads

$$\begin{aligned} A_3 = & \frac{1}{4} \left\{ (Q_o + Q_v) \left(N^2 \sum_{m_i} P_{m_i}^{(4)} + D^2 \sum_{n_i} W_{n_i}^{(4)} \right) + 2ND(Q'_s + Q'_c) \left(\frac{\eta}{\theta_4} \right)^2 \right. \\ & \left. + (R_N^2 + R_D^2)(Q_o - Q_v) \left(\frac{2\eta}{\theta_2} \right)^2 + 2R_N R_D (-O_4 S_4 - C_4 O_4 + V_4 C_4 + S_4 V_4) \left(\frac{\eta}{\theta_3} \right)^2 \right\}. \end{aligned} \quad (138)$$

In (138), the $9\bar{5}$ (ND) term is similar to the corresponding one in the supersymmetric T^4/Z_2 orientifold, but with fermions of flipped 6d chirality, the $9\bar{5}$ term with orbifold insertion $R_N R_D$ is nonsupersymmetric (but does not contribute to the vacuum energy since twisted tadpoles ask for $R_N = R_D = 0$, see below), while the other terms in (138) are precisely the supersymmetric ones.

Finally, the Möbius amplitude describes the propagation between branes and orientifold planes (holes and crosscaps). All D9-O9₊ terms, the D $\bar{5}$ -O5_– terms in the R-R sector and the D $\bar{5}$ -O9₊ terms in the NS-NS sector are as in the standard T^4/Z_2 orientifold, while the signs of all D9-O5_– terms, of the D $\bar{5}$ -O5_– terms in the NS-NS sector and of the D $\bar{5}$ -O9₊ terms in the R-R sector are inverted. In particular, this implies that the Möbius amplitude breaks supersymmetry at tree level in the D $\bar{5}$ sector, an effect felt by all open-strings ending on the D $\bar{5}$ branes. The direct (open string) Möbius amplitude is

$$M_3 = -\frac{1}{4} \left\{ N \sum_{m_i} P_{m_i}^{(4)} (\hat{O}_4 \hat{V}_4 + \hat{V}_4 \hat{O}_4 - \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) - D \sum_{n_i} W_{n_i}^{(4)} (\hat{O}_4 \hat{V}_4 + \hat{V}_4 \hat{O}_4 + \hat{S}_4 \hat{S}_4 + \hat{C}_4 \hat{C}_4) \right. \\ \left. - N (\hat{O}_4 \hat{V}_4 - \hat{V}_4 \hat{O}_4 - \hat{S}_4 \hat{S}_4 + \hat{C}_4 \hat{C}_4) \left(\frac{2\hat{\eta}}{\hat{\theta}_2} \right)^2 + D (\hat{O}_4 \hat{V}_4 - \hat{V}_4 \hat{O}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) \left(\frac{2\hat{\eta}}{\hat{\theta}_2} \right)^2 \right\}, \quad (139)$$

and parametrizing the Chan-Paton charges as $N = n_1 + n_2$, $D = d_1 + d_2$, $R_N = n_1 - n_2$, $R_D = d_1 - d_2$, the RR tadpole conditions $N = D = 32$, $R_N = R_D = 0$ determine the gauge group $[SO(16) \times SO(16)]_9 \times [USp(16) \times USp(16)]_{\bar{5}}$.

The 99 spectrum is supersymmetric, and comprises the (1,0) vector multiplets for the $SO(16) \times SO(16)$ gauge group and a hypermultiplet in the representations $(\mathbf{16}, \mathbf{16}, \mathbf{1}, \mathbf{1})$ of the gauge group. On the other hand, the $\bar{5}\bar{5}$ DD spectrum is not supersymmetric, and contains, aside from the gauge vectors of $[USp(16) \times USp(16)]$, quartets of scalars in the $(\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{16})$, right-handed Weyl fermions in the $(\mathbf{1}, \mathbf{1}, \mathbf{120}, \mathbf{1})$ and in the $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{120})$, and left-handed Weyl fermions in the $(\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{16})$. Finally, the ND sector is also non supersymmetric, and comprises doublets of scalars in the $(\mathbf{16}, \mathbf{1}, \mathbf{1}, \mathbf{16})$ and in the $(\mathbf{1}, \mathbf{16}, \mathbf{16}, \mathbf{1})$, together with additional (symplectic) Majorana-Weyl fermions in the $(\mathbf{16}, \mathbf{1}, \mathbf{16}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{16})$. These Majorana-Weyl fermions are a peculiar feature of six-dimensional spacetime, where the fundamental Weyl fermion, a pseudoreal spinor of $SU^*(4)$, can be subjected to an additional Majorana condition, if this is supplemented by a conjugation in a pseudoreal representation [86]. In this case, this is indeed possible, since the ND fermions are valued in the fundamental representation of $USp(16)$.

From the D9 brane point of view, the diagonal combination of the two $USp(16)_{\bar{5}}$ gauge groups acts as a global symmetry. This corresponds to having complex scalars and symplectic Majorana-Weyl fermions in the representations $16 \times [(\mathbf{16}, \mathbf{1}) + (\mathbf{1}, \mathbf{16})]$ of the D9 gauge group. As a result, the bose-fermi degenerate ND spectrum looks effectively supersymmetric, and indeed all $9\bar{5}$ terms do not contribute to the vacuum energy. However, as in the 6D temperature breaking discussed in [19], the chirality of the fermions in Q'_s is not the one required by 6D supersymmetry. This chirality flip is a peculiar feature of models with branes and antibranes.

Brane-antibrane interactions have been discussed recently in the literature in the context of stable non-BPS systems [26]. Our results for the D9-D $\bar{5}$ system, restricted to the open sector, provide particular examples of Type-I vacua including non-BPS stable configurations of BPS branes with vanishing interaction energy for all radii, as can be seen from the vanishing of the ND annulus amplitude.

The breaking of supersymmetry gives rise to a vacuum energy localized on the D $\bar{5}$ branes, and thus to a tree-level potential for the NS moduli, that can be extracted from the corresponding uncanceled NS tadpoles. A simple inspection shows that the only non-vanishing ones correspond to the NS characters $V_4 O_4$ and $O_4 V_4$ associated to the 6D dilaton ϕ_6 and to the internal volume v :

$$\frac{2^{-5}}{4} \left\{ \left((N - 32)\sqrt{v} + \frac{D + 32}{\sqrt{v}} \right)^2 V_4 O_4 + \left((N - 32)\sqrt{v} - \frac{D + 32}{\sqrt{v}} \right)^2 O_4 V_4 \right\}. \quad (140)$$

Using factorization and the values $N = D = 32$ needed to cancel the RR tadpoles, the potential (in the string frame) is:

$$V_{\text{eff}} = c \frac{e^{-\phi_6}}{\sqrt{v}} = c e^{-\phi_{10}} = \frac{c}{g_{\text{YM}}^2}, \quad (141)$$

where ϕ_{10} is the 10D dilaton, that determines the Yang-Mills coupling g_{YM} on the D $\bar{5}$ branes, and c is some *positive* numerical constant. The potential (141) is clearly localized on the D $\bar{5}$ branes, and is positive. This can be understood by noticing that the $O9_+$ plane

contribution to vacuum energy is negative and exactly cancels for $N = 32$. This fixes the $D\bar{5}$ brane contribution to the vacuum energy, that is thus positive, consistently with the interpretation of this mechanism as global supersymmetry breaking. The potential (141) has the usual runaway behavior in the dilaton field, as expected by general arguments.

- Brane Supersymmetry Breaking II: brane-antibrane pairs

In the previous example, the breaking of supersymmetry on the antibranes is directly enforced by the consistency of the model, which contains D9 branes and $D\bar{5}$ antibranes, a (non-BPS) stable configuration without tachyons. Somewhat different scenarios have been recently proposed in [57, 24, 25]. In the resulting models, a supersymmetric open sector is deformed allowing for the simultaneous presence of branes and antibranes of the same type. Whereas tadpole conditions only fix the total RR charge, the option of saturating it by a single type of D-brane, whenever available, stands out as the only one compatible with space-time supersymmetry. However, if one relaxes this last condition, there are no evident obstructions to considering vacuum configurations where branes and antibranes with a fixed total RR charge are simultaneously present.

Branes and antibranes of the same type are mutually interacting systems. The brane-antibrane vacuum energy in 10d, for concreteness, can be summarized by comparing the corresponding annulus amplitude with the usual brane-brane one

	open channel	closed channel
brane – brane :	$V_8 - S_8$	$V_8 - S_8$,
brane – antibrane :	$O_8 - C_8$	$V_8 + S_8$.

(142)

In the closed channel, the sign change in the RR (S_8) term simply reflects the flipped (positive) RR charge of antibranes corresponding to branes. In the open channel, this reflects into the propagation of a charged open string tachyon (O_8) and of a fermion (C_8) of opposite chirality compared to the brane-brane spectrum.

The rules for constructing this wider class of models can be simply presented referring to a ten-dimensional example [57, 25] that requires an open sector with a *net* number of 32 (anti)branes in order to cancel the resulting RR tadpole. The closed part in these models is the usual supersymmetric one. The open amplitudes, on the other hand, are

$$\begin{aligned} A_4 &= \frac{N_+^2 + N_-^2}{2} (V_8 - S_8) + N_+ N_- (O_8 - C_8) , \\ M_4 &= \pm \frac{1}{2} (N_+ + N_-) \hat{V}_8 + \frac{1}{2} (N_+ - N_-) \hat{S}_8 , \end{aligned} \tag{143}$$

where N_+ and N_- count the total numbers of D9 and $D\bar{9}$ branes. The strings stretched between D9- $D\bar{9}$ branes, with Chan-Paton factor $N_+ N_-$ in the annulus, reflect the opposite GSO projections for open strings stretched between two D-branes of the same type (99 or $\bar{9}\bar{9}$) and of different types (9 $\bar{9}$ or $\bar{9}9$) [87, 26]. While the former yields the supersymmetric Type I spectrum, the latter eliminates the vector and its spinorial superpartners, and retains the tachyon and the spinor of opposite chirality. As a result, supersymmetry is broken and an instability, signaled by the presence of the tachyonic ground state, emerges. The Möbius amplitude now involves naturally an undetermined sign for V_8 , whose tadpole is generally incompatible with the one of S_8 , and is to be relaxed. Together with A , the two signs lead to symplectic or orthogonal gauge groups with S_8 fermions in (anti)symmetric representations and tachyons and C_8 fermions in bi-fundamentals. The minus sign corresponds to $O9_+$ planes and the plus sign to $O9_-$ planes. In this last case, however, N_+ describes antibranes while N_- describes branes.

In these ten-dimensional models, the only way to eliminate the tachyon consists in introducing only D9-branes (or only $D\bar{9}$ branes). Depending on the signs in the Möbius amplitude, one thus recovers either the $SO(32)$ superstring or the $USp(32)$ model of [57].

On the other hand, more can be done if one compactifies the theory on some internal manifold. In this case, one can introduce Wilson lines (or, equivalently, separate the branes) in such a way that in the open strings stretched between separated D9 and $D\bar{9}$ branes the tachyon becomes massive. It is instructive to analyze in detail the simple case of circle

compactification. A Wilson line $W = (I_{N_+}, -I_{N_-})$ affects the annulus amplitude, that in the direct-channel now reads

$$A_4 = \frac{N_+^2 + N_-^2}{2} (V_8 - S_8) \sum_m P_m + N_+ N_- (O_8 - C_8) \sum_m P_{m+1/2} , \quad (144)$$

where $P_{m+1/2}$ denotes a sum over $\frac{1}{2}$ -shifted momentum states. In this case both the tachyon and the C_8 spinor are lifted in mass. The open sector is completed by the Möbius amplitude

$$M_4 = \frac{1}{2} \left[\pm(N_+ + N_-) \hat{V}_8 + (N_+ - N_-) \hat{S}_8 \right] \sum_m P_m \quad (145)$$

and at the massless level comprises gauge bosons in the adjoint of $\text{SO}(N_+) \times \text{SO}(N_-)$ (or $\text{USp}(N_+) \times \text{USp}(N_-)$, depending on the sign of \hat{V}_8 in M) and S spinors in (anti)symmetric representations. We display here the effective action of branes/O-planes with supergravity fields in the case of a minus sign in (145)

$$S = \int d^{10}x \{ \sqrt{G} \mathcal{L}_{SUGRA} - (N_+ - 16) T_8 (\sqrt{G} e^{-\Phi} + A_9) \delta(y) - T_8 [\sqrt{G} (N_- - 16) e^{-\Phi} - (N_- + 16) A_9] \delta(y - \pi R) \} , \quad (146)$$

where \mathcal{L}_{SUGRA} is the 10d supergravity lagrangian. Notice in (146) the peculiar interaction of $\text{D}\bar{8}$ antibranes with O-planes.

Interestingly enough, it was recently realized [25] that the sign flip of the RR charge of the O5 plane in some 4d orientifold models with no supersymmetric solution ($Z_2 \times Z_2$ with discrete torsion or Z_4) defines consistent models with supersymmetry on branes and broken supersymmetry on antibranes. Indeed, for example in the Z_4 model the supersymmetric construction leads to a tadpole in the Klein bottle (of the type $1/V_3$) which cannot be cancelled by the existing (D9 and D5) branes in the model. By changing the charge of the O5 plane (or, equivalently, the Ω projection in the closed string Z_2 twisted sector), this tadpole becomes massive and the open spectrum can be consistently constructed without further obstructions. Another interesting feature of brane-antibrane systems is the presence of mutual forces. It was suggested in [24] and was explicitly shown in [25], that by

adding D9-D $\bar{9}$ and D5-D $\bar{5}$ pairs, scalar potentials are generated by the NS-NS tadpoles such that some or all radii of the compact space are stabilized. These models present some phenomenological interest, as will be seen later in this review. A generic feature of all models with supersymmetry broken on a collection of (anti)branes, however, is that there is a dilaton tadpole, which means that the correct background is not the Minkowski one with maximal symmetry. Identifying the correct background is therefore an important step in unravelling the properties of these models. A step forward was recently made in [58], where a background (with $SO(9)$ symmetry) of the 10d Type I model with gauge group $USp(32)$ was found. The tenth coordinate turns out to be spontaneously compactified, so that the length of the tenth dimension is finite. The geometry of the background is $R^9 \times S^1/Z_2$, with the zero-mode of the graviton localized near one of the boundaries of the interval.

All the models with broken supersymmetry discussed in this Section face the problem of the cosmological constant [88]. It seems difficult to find models with naturally zero (or very small) vacuum energy. There exist however explicit perturbative Type II examples and also Type I descendants for the class of models with supersymmetry breaking by compactification [89]. To date, there are no similar models exhibiting the phenomenon of brane supersymmetry breaking.

8. Millimeter and TeV $^{-1}$ large extra dimensions

The presence of branes in Type I, Type II, Type O strings and M-theory opens new perspectives for particle physics phenomenology. Indeed, we already saw in (13) that in Type I strings the string scale is not necessarily tied to the Planck scale. In view of the new D-brane picture, let us take a closer look at the simplest example of compactified Type I string, with only D9 branes present. We found in (13) that the string scale can be in the TeV range if the string coupling is extremely small, $\lambda_I \sim 10^{-32}$. Then, from the second relation (12) one can see that in this case the compact volume is very small

$VM_I^6 \sim 10^{-32}$. Let us split the compact volume into two parts, $V = V^{(1)}V^{(2)}$, where $V^{(1)}$, of dimension $6 - n$, is of order one in string units and $V^{(2)}$, of dimension n , is very small. The Kaluza-Klein states of the brane fields along $V^{(2)}$ are much heavier than the string scale and therefore are difficult to excite. The physics is then better captured in this case performing T-dualities along $V^{(2)}$, which read

$$\lambda_I' = \frac{\lambda_I}{V^{(2)}M_I^n} \quad , \quad V_\perp = \frac{1}{V^{(2)}M_I^{2n}} \quad . \quad (147)$$

In the T-dual picture, neglecting numerical factors, the relations (147) become

$$M_P^2 \sim \frac{1}{\alpha_{GUT}\lambda_I'} V_\perp M_I^{2+n} \quad , \quad \frac{1}{\alpha_{GUT}} \sim \frac{V_\parallel M_I^{6-n}}{\lambda_I'} \quad , \quad (148)$$

where for transparency of notation we redefined $V^{(1)} \equiv V_\parallel$. After the n T-dualities, the D9 brane becomes a D(9-n) brane, since the T-dual winding modes of the bulk (orthogonal) compact space are very heavy and therefore the brane fields cannot propagate in the bulk. As seen from (148), for a very large bulk volume the string scale can be very low $M_I \ll M_P$. The geometric picture here is that we have a D-brane with some compact radii parallel to it, of order M_I^{-1} , and some very large, orthogonal compact radii. In particular, if the full compact space is orthogonal to the brane ($n = 6$), from (148) the T-dual string coupling is fixed by the unified coupling $\lambda_I' \sim \alpha_{GUT}$, and therefore we find [38]

$$M_P^2 \sim \frac{1}{\alpha_{GUT}^2} V_\perp M_I^{2+n} \quad , \quad (149)$$

a relation similar to that proposed in the field-theoretical scenario of [36].

Let us now imagine a “brane-world” picture¹⁵, in which the Standard Model gauge group and charged fields are confined to the D-brane under consideration. We can then ask a very important question: what are the present experimental limits on parallel R_\parallel and perpendicular R_\perp type radii ? The Standard Model fields have light KK states in the parallel directions R_\parallel . Their possible effects in accelerators were studied in detail

¹⁵For earlier proposals of such a “brane-world” picture, see [90].

[81], and the present limits are $R_{\parallel}^{-1} \geq 4 - 5$ TeV. On the other hand, Standard Model excitations related to R_{\perp} are very heavy and are thus irrelevant at low energy. The main constraints on R_{\perp} come from the presence of very light winding (KK after T-dualities) gravitational excitations, which can therefore generate deviations from the Newton law of gravitational attraction. The actual experimental limits on such deviations are limited to the cm range and experiments in the near future are planned to improve them [91]. For $M_I \sim \text{TeV}$ in (149), the case of only one extra dimension is clearly excluded, since it asks for $R_{\perp}^{-1} \sim 10^8$ Km. However, for two extra dimensions, we find $R_{\perp}^{-1} \sim 1\text{mm}$, not yet excluded by the present experimental data. On the other hand, if all compact dimensions are perpendicular and large, one finds $R_{\perp}^{-1} \sim \text{fm}$, distance scale completely inaccessible for Newton law measurements. Such a physical picture with $M_I \sim \text{TeV}$ provides in principle a new solution to the gauge hierarchy problem, i.e. of why the Higgs mass M_h is much lower than the 4d Planck mass M_P , provided the physical cutoff M_I has similar values $M_I \simeq M_h$.

In Type I strings, the brane we considered can be a D9 or a D5 brane, up to T-dualities. Our brane world can live on any of the branes; let us choose for concreteness that our Standard Model gauge group be on a D9 brane. Notice that, while D9 branes fill (before T-dualities) the full 10d space, D5 branes fill only six dimensions. The D5 degrees of freedom can of course propagate in what we called previously bulk space, and can change slightly our previous picture. The relation between the corresponding D9 and D5 gauge couplings is

$$g_9^2/g_5^2 = V_{\perp} \quad , \quad (150)$$

where V_{\perp} denotes here (before T-dualities) the compact volume perpendicular to the D5 brane. If $V_{\perp} \gg 1$ in string units, then D5 branes live in (at least part of) the bulk and, by (150) their gauge coupling is very suppressed compared to our (D9) gauge coupling. In particular, if V_{\perp} in (150) is as in (149), the D5 gauge couplings are of gravitational strength. The fields in mixed 95 representations are charged under both gauge groups. Then, due to their very small gauge couplings, the D5 gauge groups manifest themselves

as global symmetries on our D-brane, and could be used for protecting baryon and lepton number nonconservation processes. Indeed, global symmetries are presumably violated by nonrenormalizable operators suppressed by the fundamental scale M_I and, since M_I can be very low, we need suppression of many higher-dimensional operators.

There are clearly many challenging questions that such a scenario must answer [92] in order to be seriously considered as an alternative to the conventional “desert picture” of supersymmetric unification at energies of the order of 10^{16} GeV. The gauge hierarchy problem still has a counterpart here, understanding the possible mm size of the compact dimensions (perpendicular to our brane) in a theory with a fundamental length (energy) in the 10^{-16} mm (TeV) range. There are several ideas concerning this issue in the literature [93], which however need further studies in realistic models in order to prove their viability. A serious theoretical question concerns gauge coupling unification, that in this case, if it exists, must be completely different from the conventional MSSM (Minimal Supersymmetric Standard Model) one. Moreover, there is more and more convincing evidence for neutrino masses and mixings, and the conventional picture provides an elegant explanation of their pattern via the seesaw mechanism [99] with a mass scale of the order of the $10^{12} - 10^{15}$ GeV, surprisingly close to the usual GUT scale. From this viewpoint, neutrino masses can be considered as the first experimental manifestation of physics beyond the Standard Model (see for example [100]). The new scenario described above should therefore provide at least a qualitative picture for neutrino masses and mixings. Cosmology, astrophysics [36, 94], accelerator physics [81, 101] and flavor physics [37, 95] put additional strong constraints on the low-scale string scenario.

9. Gauge coupling unification

Models with gauge-coupling unification at low energy triggered by Kaluza-Klein states were independently proposed in [37], at the same time as brane-world models with a low-

string scale. Both provide possible solutions to the gauge hierarchy problem. It is transparent, however, that low-scale string models are the natural framework for this fast-driven unification. In this chapter we separate the discussion into two steps: we begin with the field-theoretic picture originally proposed in [37], and then move to the Type I string approach developed in [50, 70] which brings some new, interesting features.

- Field theory approach

The essential ingredient in this approach are the KK excitations of the Standard Model gauge bosons and matter multiplets and their contribution to the energy evolution of the physical gauge couplings. In the early paper [103], Taylor and Veneziano pointed out that the KK excitations give power-law corrections that at low energy can be interpreted as threshold corrections. Actually, as shown in [37], if the energy is higher than the KK compactification scale $1/R$, these corrections should be interpreted as power-law accelerated evolutions of gauge couplings that, under some reasonable assumptions, can bring these couplings together at low energies.

The one-loop evolution of gauge couplings in 4d between energy scales μ_0 and μ can be computed with standard methods, and the final result can be cast into the form

$$\frac{1}{\alpha_a(\mu)} = \frac{1}{\alpha_a(\mu_0)} + \frac{1}{2\pi} \sum_r \text{Str} \int_{1/\mu^2}^{1/\mu_0^2} \frac{dt}{t} Q_{a,r}^2 \left(\frac{1}{12} - \chi_r^2 \right) e^{-tm_r^2} , \quad (151)$$

where $Q_{a,r}$ is the gauge group generator in the representation r of the gauge group, m_r^2 is the mass operator and χ_r is the helicity of various charged particles contributing in the loop. In 4d (151) can be readily integrated as usual in order to obtain, for example

$$\frac{1}{\alpha_a(\mu)} = \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi} \ln \frac{\mu}{M_Z} , \quad (152)$$

where b_a are the beta-function coefficients defined as in (123) for a supersymmetric theory.

Let us start, for reasons to be explained later on, with the MSSM in 4d and try to extend it to 5d, where the fifth dimension is a circle of radius $R_{||}$, in the notation introduced in

the previous section. In this case (151) generalizes to

$$\frac{1}{\alpha_a(\mu)} = \frac{1}{\alpha_a(\mu_0)} + \frac{1}{2\pi} \sum_r \text{Str} \int_{1/\mu^2}^{1/\mu_0^2} \frac{dt}{t} Q_{a,r}^2 \left(\frac{1}{12} - \chi_r^2 \right) \left(\sum_n e^{-tm_{n,r}^2(R_{||})} + e^{-tm_r^2} \right), \quad (153)$$

where we separated the mass operator into a part containing fields with KK modes and a part containing fields without KK modes. Indeed, consider again for concreteness gauge couplings of a D9 brane and consider δ large compact dimensions $R_{||}M_I \gg 1$ parallel to D9 and orthogonal to D5. Then the 99 states will have associated KK states, but 95 states will not. Evaluating (153) with $\mu_0 = M_Z$, one finds

$$\begin{aligned} \frac{1}{\alpha_a(\mu)} &= \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi} \ln \frac{\mu}{M_Z} - \frac{\tilde{b}_a}{2\pi} \int_{1/\mu^2}^{1/M_Z^2} \frac{dt}{t} \theta_3^\delta \left(\frac{it}{\pi R_{||}^2} \right) \\ &\simeq \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi} \ln \frac{\mu}{M_Z} + \frac{\tilde{b}_a}{2\pi} \ln(\mu R_{||}) - \frac{\tilde{b}_a}{2\pi} [(\mu R_{||})^\delta - 1]. \end{aligned} \quad (154)$$

The coefficients \tilde{b}_a in (154) denote one-loop beta-function coefficients of the massive KK modes, to be computed in each specific model. The important term contained in (154) is the power-like term $(\mu R_{||})^\delta \gg 1$, which overtakes the logarithmic terms in the higher-dimensional regime and governs the eventual unification pattern.

The power-like term is proportional to the coefficients \tilde{b}_a , that *are not* the usual 4d MSSM ones which successfully predict unification. Therefore, from this point of view the MSSM unification would just be an accident, and this fact is disappointing. Let us however go on and find the *minimal* possible embedding of the MSSM in a 5d spacetime. Before doing it, notice that compactifying on a circle a supersymmetric theory in 5d gives a 4d theory with at least $\mathcal{N} = 2$ supersymmetries. The simplest way to avoid this is to compactify on an *orbifold*, a singular space defined in Section 5. We consider as example the case of a Z_2 orbifold which breaks supersymmetry down to $\mathcal{N} = 1$. 5d fields can be even or odd under this operation, in particular 5d Dirac fermions in 4d truncate into one even Weyl fermion containing a zero mode and its KK tower and one odd Weyl fermion, with no associated zero mode, and its KK tower. It is easy to realize that a 4d chiral multiplet (ψ_1, ϕ_1) can arise from a 5d hypermultiplet containing KK modes $(\psi_1^{(n)}, \psi_2^{(n)}, \phi_1^{(n)}, \phi_2^{(n)})$ or

from a 5d vector multiplet. Similarly, a 4d massless vector multiplet (λ, A_μ) arises from a 5d vector multiplet containing the KK modes $(\lambda^{(n)}, \psi_3^{(n)}, A_\mu^{(n)}, a^{(n)})$, where $\psi_i^{(n)}$, $i = 1, 2, 3$ are 4d Weyl fermions and $\phi_i^{(n)}$, $a^{(n)}$ are complex scalars. The massive KK representations are clearly nonchiral, while chirality is generated at the level of zero modes.

The simplest embedding of the MSSM in 5d is the following [37]. The gauge bosons and the two Higgs multiplets of MSSM are already in real representations of the gauge group and naturally extend to KK representations $(\lambda^{(n)}, \psi_3^{(n)}, A_\mu^{(n)}, a^{(n)})$ and $(\psi_1^{(n)}, \psi_2^{(n)}, H_1^{(n)}, H_2^{(n)})$, respectively¹⁶. The matter fermions of MSSM, being chiral, can either contain only zero modes or, alternatively, can have associated mirror fermions and KK excitations for $\eta = 0, 1, 2, 3$ families. The unification pattern does not depend on η (the value of the unified coupling, on the other hand, does), since each family forms a complete $SU(5)$ representation. The massive beta-function coefficients for this simple 5d extension of the MSSM are

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = \left(\frac{3}{5}, -3, -6\right) + \eta(4, 4, 4) , \quad (155)$$

where, as usual, we use the $SU(5)$ embedding $\tilde{b}_1 \equiv (3/5)\tilde{b}_Y$. These coefficients in the case $\eta = 3$ are not the same as the MSSM ones $(b_1, b_2, b_3) = (33/5, 1, -3)$. However, interestingly enough, as seen from Figure 2, the couplings unify with a surprisingly good precision, for any compact radius $10^3 \text{ GeV} \leq R_{||}^{-1} \leq 10^{15} \text{ GeV}$, at a energy scale roughly a factor of 20 above the compactification scale $R_{||}^{-1}$. The algebraic reason for this is that, in order to have MSSM unification, the conditions that must be fulfilled are

$$\frac{B_{12}}{B_{13}} = \frac{B_{13}}{B_{23}} = 1 \quad , \text{ where } B_{ac} \equiv \frac{\tilde{b}_a - \tilde{b}_c}{b_a - b_c} . \quad (156)$$

Although these relations are not satisfied exactly in our case, they are nonetheless approximately satisfied

$$\frac{B_{12}}{B_{13}} = \frac{72}{77} \simeq 0.94 \quad , \quad \frac{B_{13}}{B_{23}} = \frac{11}{12} \simeq 0.92 . \quad (157)$$

¹⁶As one of the two Higgses in a hypermultiplet is odd under Z_2 , the simplest extension actually has one KK Higgs hypermultiplet and one Higgs without KK excitations.

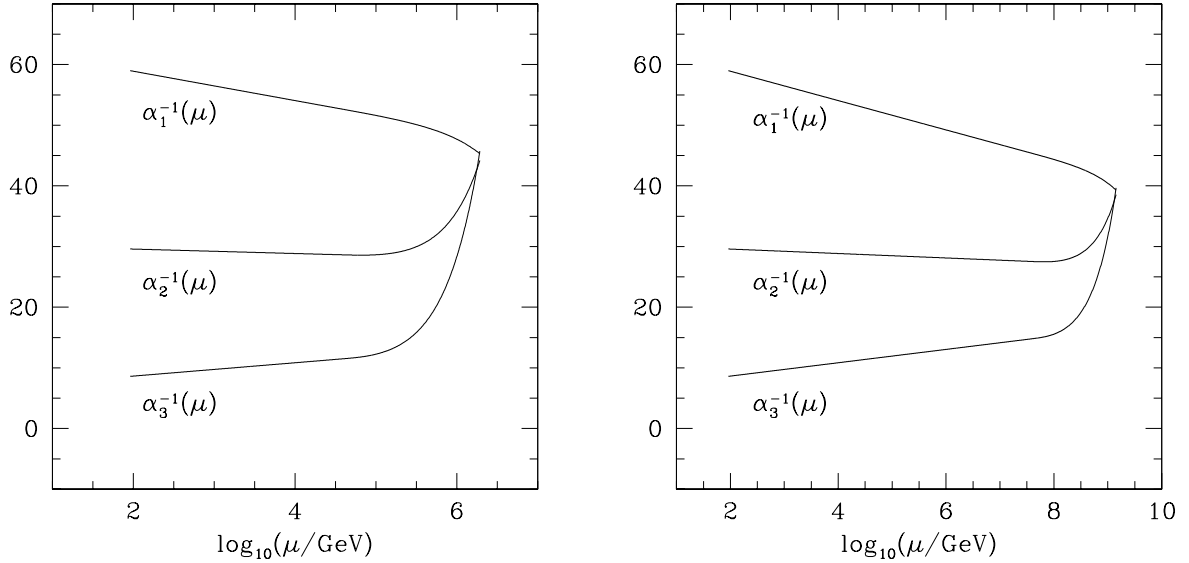


Figure 2: Unification of gauge couplings in the presence of extra spacetime dimensions. We consider two representative cases: $R^{-1} = 10^5 \text{ GeV}$ (left), $R^{-1} = 10^8 \text{ GeV}$ (right). In both cases we have taken $\delta = 1$ and $\eta = 0$.

This fast unification with KK states is another numerical miracle, similar to the MSSM unification and may be regarded as one serious hint pointing into the possible relevance of extra dimensions in our world. There are clearly a lot of questions that this scenario can raise, which were discussed in detail in the literature [37, 104, 105], the most important ones being:

- The perturbative nature.

Indeed, even if unified coupling in Figure 2 is about $\alpha_{GUT} \simeq 1/50$ for $M_{GUT} \sim 10 \text{ TeV}$, the parameter controlling the loop expansion is $N_{KK}\alpha_{GUT}$, where N_{KK} is here the number (of order 20) of KK states with masses higher than the unification scale M_{GUT} . This parameter is large (of order $2/5$), and therefore perturbativity seems to be lost. However, things are slightly better than expected. Indeed, massive modes come into $\mathcal{N} = 2$ multiplets. In $\mathcal{N} = 2$ theories beta functions get contributions only at one loop. Therefore

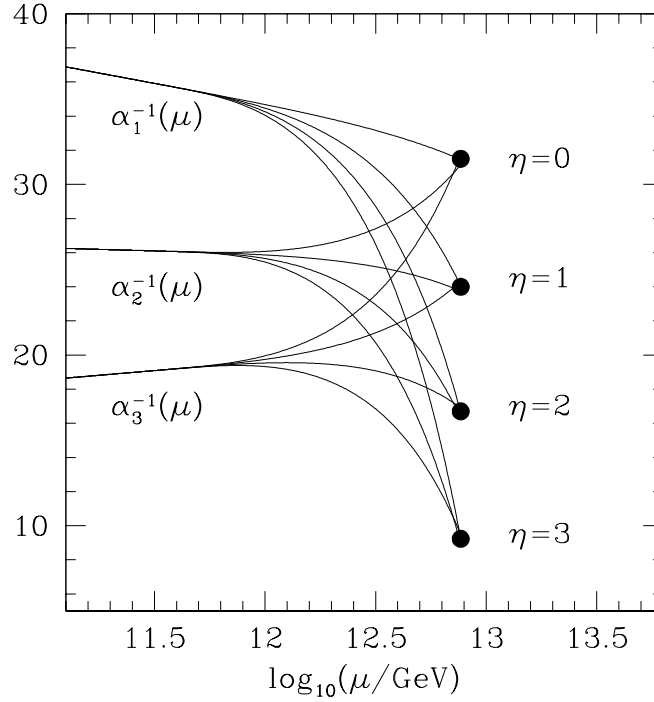


Figure 3: Unification of gauge couplings in the presence of extra spacetime dimensions. Here we fix $R^{-1} = 10^{12}$ GeV, $\delta = 1$, and we vary η . For this value of R^{-1} , we see that the unification remains perturbative for all η .

the higher-order loops must contain zero-mode propagators, which have reduced $\mathcal{N} = 1$ supersymmetry but have no KK modes. For example, the (two loop)/(one-loop) effects are naively of order $N_{KK}\alpha_{GUT}$ but, due of the argument above, they are actually only of order α_{GUT} . Two-loop contributions induced by Yukawa coupling corrections could in principle be larger, but actually they are still under control (see, for example, M. Masip in [104]). Notice that the most perturbative case is $\eta = 0$, and increasing η from 0 to 3 renders the model less and less perturbative (see Figure 3).

- The sensitivity to high-energy thresholds

The result of the computation is more sensitive to high-energy thresholds than the MSSM result is, if we consider the unification in the sense of a Grand Unified Theory. A

related question is the suppression of baryon number violating operators, in low-scale strings, which are induced by states of the GUT theory. If we consider the unification in the string sense, as will be shown in the next paragraph string thresholds affect only the $\mathcal{N} = 1$ sectors of the theory, while in $\mathcal{N} = 2$ sectors, responsible for the power-law evolution, the string states decouple [74, 50] and the corrections come only from KK massive states. The running stops at a higher winding scale, without the need of a GUT gauge group and new thresholds there.

- The need for supersymmetry

We considered above a higher-dimensional extension of MSSM. We could in principle try a similar extension of the Standard Model, without invoking supersymmetry. In this case the unification is still possible [37], at the price of introducing additional gauge group representations. The extension is consequently not minimal by any means. Therefore, it is amusing that even in case of large extra dimensions, supersymmetry seems to play a role in the unification of gauge couplings.

In order to have a physical interpretation of the unification scale discussed above, we now turn to the string approach, using results derived in Section 6.

- String theory approach

In a superstring model, the threshold corrections to gauge couplings (116) can be generically written as

$$\mathcal{B}_a = \mathcal{B}_a^{(\mathcal{N}=4)} + \mathcal{B}_a^{(\mathcal{N}=2)} + \mathcal{B}_a^{(\mathcal{N}=1)} , \quad (158)$$

where the different terms in the rhs of (158) denote contributions from $\mathcal{N} = 4$, $\mathcal{N} = 2$ and $\mathcal{N} = 1$ sectors, respectively. The $\mathcal{N} = 4$ sectors, containing the full $\Gamma^{(6)}$ lattice in the notation of Section 3, have a 10d origin and give no contribution to threshold corrections. The $\mathcal{N} = 2$ sectors contain the lattice of one compact torus $\Gamma^{(2)}$. In these sectors only BPS KK states contribute to threshold corrections and string oscillators decouple [74, 50].

Therefore, their contribution to the evolution of gauge couplings does not stop at the string scale M_I , but rather, as we will see, at a heavy KK scale. The $\mathcal{N} = 1$ sectors have no KK excitations and give a moduli-independent contribution to threshold-corrections, interpreted as the $\mathcal{N} = 1$ contribution to gauge couplings, running up to M_I .

The string one-loop threshold corrections coming from $\mathcal{N} = 2$ sectors were computed in (120). We also saw in Section 6 that, in addition to the dilaton $\text{Im}S \sim 1/l$, there are tree-level (disk) contributions from couplings of gauge fields to the twisted moduli m_k , displayed in (106). Putting all the terms together, we find the complete one-loop gauge couplings

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{1}{l} + \sum_k s_{ak} m_k + \frac{1}{4} b_a^{(\mathcal{N}=1)} \ln \frac{M_I^2}{\mu^2} - \frac{1}{4} \sum_{i=1}^3 b_{ai}^{(\mathcal{N}=2)} \ln(\sqrt{G_i} \mu^2 |\eta(U_i)|^4 \text{Im} U_i) , \quad (159)$$

where for a rectangular torus of radii R_1, R_2 , we have $\sqrt{G_i} = R_1 R_2$ and $\text{Im} U = R_1/R_2$. In (159), $b_{ai}^{(\mathcal{N}=2)}$ denote beta function coefficients from $\mathcal{N} = 2$ sectors having KK excitations in the compact torus T^i . The total beta function (123) of the model is, in these notations,

$$b_a = b_a^{(\mathcal{N}=1)} + \sum_{i=1}^3 b_{ai}^{(\mathcal{N}=2)} . \quad (160)$$

Let us now consider the field-theory limit of the corrections given by an $\mathcal{N} = 2$ sector, depending on a torus of radii $R_{1,2}$. In the limit $R_1 \rightarrow \infty$ and R_2 fixed, the corrections are linearly divergent as $\Lambda_2 \sim R_1/R_2$. These power-law corrections can be used to lower the unification scale [37] in models with a low value of the string scale M_I . Notice that, in all the above computations, μ denoted an infrared energy scale, smaller than any KK mass scales. Actually, for energies $\mu \gg R_1^{-1}$, relevant for the $R_1 \rightarrow \infty$ limit, it can be seen that the previous factor R_1/R_2 really becomes $R_1\mu$, thus reproducing the field-theory derivation (154) with $\delta = 1$. In this case, to get unification one needs $10^3 \text{ GeV} \leq R_1^{-1} \leq 10^{15} \text{ GeV}$. On the other hand, it is at first sight surprising that in the opposite limit of very heavy KK states (windings after T-duality) $R_1 \rightarrow 0$, there is a divergent contribution $\Lambda_2 \sim R_2/R_1$. In particular, this applies to mm perpendicular dimensions and can therefore spoil the

solution to the hierarchy problem [50, 106]. These corrections can however be avoided in a class of Type I models that satisfy local tadpole cancellation in the corresponding direction [46], [19].

Another interesting and unexpected feature is that in the limit $R_1, R_2 \rightarrow \infty$ with R_1/R_2 fixed, $\Lambda_2 \sim \ln(R_1 R_2 \mu^2)$, instead of the quadratic divergence ($\delta = 2$ in (154)) expected in the field theory approach. The same result holds in the $R_1, R_2 \rightarrow 0$ limit. This result can be understood by the following argument [106]. After T-duality, the two directions are very large and perpendicular to the brane under consideration. One-loop threshold corrections can also be understood as tree-level coupling of gauge fields to closed sector fields, which have a bulk variation reproducing the threshold dependence on the compact space. The bulk variation can be computed in a supergravity approximation, solving classical field equations for closed fields coupled to various sources subject to global neutrality (or global tadpole cancellation) in the compact space. As the Green function in two dimensions has a logarithmic behaviour, this explains the logarithmic term $\ln(R_1 R_2 \mu^2)$. The same argument in one compact dimension explains the linearly divergent term previously discussed.

The logarithmic evolution $\Lambda_2 \sim \ln(R_1 R_2 \mu^2)$ can also be used to achieve unification at a high energy scale, even if the fundamental string scale has much lower values [50], by “running” beyond the string scale.

Notice that both the power-law and the logarithmic evolution of gauge couplings use $\mathcal{N} = 2$ beta-functions. As shown in the field-theory approach, a simple higher-dimensional extension succeeds in producing unification with $\mathcal{N} = 2$ sectors. In this case however, MSSM unification would be just a miraculous accident. It would be useful to see if one can obtain the usual MSSM unification in models with a low string scale. One possibility recently proposed in [107] takes advantage of the couplings to twisted fields present in (159) in models without $\mathcal{N} = 2$ sectors. Let us assume that in some models $s_{ak} = c_k b_a$ and, in addition, that the twisted fields have some vevs $\langle m_k \rangle$. In terms of $\sum_k c_k m_k \equiv c m$, the

one-loop relation (159) becomes

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{4\pi^2}{g_a^2}|_{\text{tree}} + c b_a < m > + \frac{b_a}{2} \ln \frac{M_I}{\mu} = \frac{4\pi^2}{g_a^2}|_{\text{tree}} + \frac{b_a}{2} \ln \frac{M_I e^{2c<m>}}{\mu} . \quad (161)$$

The real unification scale is therefore $M_{GUT} = \exp(2c < m >) M_I$, which, depending on the sign in the exponential, can be much larger than M_I . There are some Type I models where indeed the proportionality relation $s_{ak} = c_k b_a$ holds [108, 107]. In these models, unfortunately, $< m > = 0$ and this “mirage” unification does not occur [70]. It is still reasonable to hope that in some other models all conditions are fulfilled and that the mechanism can be implemented¹⁷. A possible scenario is the following. Suppose that gaugino condensation takes place in a gauge group factor G_a , coupling to m , giving rise to a nonperturbative superpotential for the complex chiral superfields S, M of the form

$$W(S, M) \sim e^{-\alpha(S+s_a M)} , \quad (162)$$

where α is a numerical factor depending on the gauge factor G_a and its matter content. For a large class of Kähler potentials for the moduli M and if the dilaton S is stabilized, a nonzero value $< M > \sim M_P$ is easily generated and can provide the (mirage) unification discussed above.

10. Supersymmetry breaking

- Breaking through compactification in field theory: the Scherk-Schwarz mechanism

The Scherk-Schwarz mechanism for breaking supersymmetry takes advantage of the presence of compact spaces in compactifications of higher-dimensional supersymmetric field theories or of superstrings. The main idea is to use symmetries \mathcal{S} of the higher-dimensional theory which do not commute with supersymmetry, typically R-symmetries or the fermion

¹⁷A proposal trying to combine [50] and [107] was also recently studied in [109].

number $(-1)^F$. Then, after being transported around the compact space (a circle of radius R , for concreteness and coordinate $0 \leq y \leq 2\pi R$), bosonic and fermionic fields Φ_i return to the initial value (at $y = 0$) only up to a symmetry operation

$$\Phi_i(2\pi R, x) = U_{ij}(\omega) \Phi_j(0, x) , \quad (163)$$

where the matrix $U \in \mathcal{S}$ is different for bosons and fermions and x are noncompact coordinates. At the field theory level, (163) implies that the Kaluza-Klein decomposition on the circle is changed so that zero modes acquire a nontrivial dependence on the y coordinate, according to

$$\Phi_i(y, x) = U_{ij}(\omega, y/R) \sum_m e^{\frac{imy}{R}} \Phi_j^{(m)}(x) , \quad (164)$$

where ω is a number, quantized in String Theory. The matrix U satisfies some additional constraints in supergravity in order for the generated scalar potential to be positive definite. The ansatz considered by Scherk and Schwarz is $U = \exp(My)$, where M is an antihermitian matrix. Then kinetic terms in the y direction generate mass terms and break supersymmetry, the resulting fermion-boson splittings being equal to ω/R . This twisting procedure is very similar to the breaking of supersymmetry at *finite temperature* and, because of this, the terms breaking supersymmetry are UV finite, even at the field theory level.

The mechanism can be applied in globally supersymmetric models, in supergravity models and in superstrings. The breaking is induced by the different boundary conditions for bosons and fermions and is therefore an explicit breaking. However, at the supergravity level, it appears to be *spontaneous*. In order to clarify this point, let us consider a simple global model and a local (supergravity) one.

i) a globally supersymmetric model

The model has one hypermultiplet in 5d, containing one Dirac fermion Ψ and two

complex scalars ϕ_1, ϕ_2 , described by the free lagrangian

$$\mathcal{L} = -\frac{i}{2}(\bar{\Psi}\gamma^M\partial_M\Psi - \partial_M\bar{\Psi}\gamma^M\Psi) - |\partial_M\phi_1|^2 - |\partial_M\phi_2|^2. \quad (165)$$

The model (165) has several symmetries. Let us choose the R-symmetry¹⁸ that leave the fermion invariant and rotates the two complex scalars between themselves. The Scherk-Schwarz matrix reads $M = i(\omega/R)\sigma_2$, where σ_2 is the second Pauli matrix. After the modified KK reduction (164), one finds the resulting 4d lagrangian

$$\begin{aligned} \mathcal{L} = & -\sum_m \left\{ \frac{i}{2}(\Psi_1^{(m)}\sigma^\mu\overleftrightarrow{\partial}_\mu\bar{\Psi}_1^{(m)} + \Psi_2^{(m)}\sigma^\mu\overleftrightarrow{\partial}_\mu\bar{\Psi}_2^{(m)}) + |\partial_\mu\phi_1^{(m)}|^2 + |\partial_\mu\phi_2^{(m)}|^2 \right. \\ & \left. + \frac{m}{R}(\Psi_1^{(m)}\Psi_2^{(m)} + h.c.) + \frac{m^2 + \omega^2}{R^2}(|\phi_1^{(m)}|^2 + |\phi_2^{(m)}|^2) \right\}, \end{aligned} \quad (166)$$

where we defined the Weyl fermions $\Psi^{(m),T} = (\Psi_1^{(m)}, \bar{\Psi}_2^{(m)})^T$. The lagrangian (166) describes the free propagation of massive Weyl spinors $\Psi_1^{(m)}, \Psi_2^{(m)}$, of mass m/R and the free propagation of massive complex scalars $\phi_1^{(m)}, \phi_2^{(m)}$, of mass squared $(m^2 + \omega^2)/R^2$. Supersymmetry is explicitly broken by the *soft* mass term ω^2/R^2 .

In orbifolds, there is a compatibility condition between the orbifold action θ and the Scherk-Schwarz twisting matrix $[\theta, U] = 0$. In particular, if the coordinate y in question is orbifold invariant $\theta y = y$, this implies $[\theta, M] = 0$, while if it is Z_2 twisted $\theta y = -y$, this implies $\{\theta, M\} = 0$.

ii) a local model: supersymmetry breaking in M-theory

The example we discuss now is the compactified version of supersymmetry breaking in M-theory, discussed in general terms in Section 4 and realized also in Type I strings in Section 7.

Consider the simplest truncation of 11d supergravity down to 5d, keeping only the breathing mode of the compact space $g_{i\bar{j}} = \delta_{i\bar{j}}\exp(\sigma)$, and concentrate for simplicity

¹⁸P. Fayet [79] was the first to use R-symmetries in order to produce phenomenologically interesting soft masses in global supersymmetric models. He proposed compact radii in the TeV range for this purpose.

on zero modes only. In this case, the only matter multiplet in 5d (in addition to the 5d gravitational multiplet with bosonic fields (g_{MN}, C_M) , where $C_{M\bar{i}\bar{j}} = (1/6)A_M\delta_{\bar{i}\bar{j}}$ is a vector field originating from the 3-form of 11d supergravity) is the universal hypermultiplet of bosonic fields (σ, C_{MNP}, a) , with $C_{ijk} = (1/6)\epsilon_{ijk}a$, whose scalar fields parametrize the coset $SU(2, 1)/SU(2) \times U(1)$ [110]. The bosonic 5d supergravity lagrangian is¹⁹

$$\begin{aligned} \mathcal{S}_5 = & \frac{1}{2k_5^2} \int d^5x \sqrt{g} \{ R - \frac{9}{2}(\partial_M \sigma)^2 - \frac{1}{24} e^{6\sigma} G_{MNPQ} G^{MNPQ} - \frac{3}{2} F_{MN} F^{MN} - 2 e^{-6\sigma} |\partial_M a|^2 \} \\ & - \frac{1}{k_5^2} \int d^5x \epsilon^{MNPQR} \{ \frac{i}{\sqrt{2}} C_{MNP} \partial_Q a \partial_R a^\dagger + \frac{1}{2\sqrt{2}} A_M F_{NP} F_{QR} \} , \end{aligned} \quad (167)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$. The compactification from 5d to 4d is on the orbifold S^1/Z_2^{HW} , with orbifold action Z_2^{HW} defined in (63) of Section 4. The lagrangian of the universal hypermultiplet can be derived from the 4d Kähler potential [110]

$$\mathcal{K} = -\ln (S + S^\dagger - 2a^\dagger a) , \quad (168)$$

lifted back to 5d, where $S = \exp(3\sigma) + a^\dagger a + ia_1$ and the axion a_1 is defined by the Hodge duality $\sqrt{2} \exp(6\sigma) G_{MNPQ} = \epsilon_{MNPQR} (\partial^R a_1 + ia^\dagger \overleftrightarrow{\partial}^R a)$. The lagrangian (167) has a global $SU(2)_R$ symmetry, acting linearly on the redefined hypermultiplet fields

$$z_1 = \frac{1-S}{1+S} \quad , \quad z_2 = \frac{2a}{1+S} , \quad (169)$$

which form a doublet (z_1, z_2) . In the gravitational multiplet, the 5d Dirac gravitino is equivalent to two 4d Majorana gravitinos, transforming as an $SU(2)_R$ doublet. One of the gravitini is even under Z_2^{HW} and has a zero mode (before the Scherk-Schwarz twisting), while the other is odd and has only massive KK excitations. The Z_2^{HW} projection acts on the hypermultiplet as $Z_2^{HW} S = S$, $Z_2^{HW} a = -a$, which translates on the $SU(2)$ doublet in the obvious way

$$Z_2^{HW} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} . \quad (170)$$

¹⁹There are also terms coming from the modified Bianchi identity (65) correcting the lagrangian (167).

We neglect them here for simplicity, but they can be found in [33], for example.

The Horava-Witten projection then asks for using the $U(1)_R$ subgroup of $SU(2)_R$ and the corresponding Scherk-Schwarz decomposition reads [32]

$$\begin{pmatrix} \hat{z}_1 \\ \hat{z}_2 \end{pmatrix} = \begin{pmatrix} \cos M_0 x_5 & \sin M_0 x_5 \\ -\sin M_0 x_5 & \cos M_0 x_5 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad (171)$$

corresponding to the matrix $M = iM_0\sigma_2$. Notice that, thanks to the anticommutation relation $\{Z_2^{HW}, M\} = 0$, the fields \hat{z}_i have the same Z_2^{HW} parities as the fields z_i . The 4d complex superfields of the model are S (with the zero mode $a = 0$) and T , where $T = g_{55} + i C_5$ and the axion C_5 is the fifth component of the vector field in the 5d gravitational multiplet. The resulting scalar potential in 4d in the Einstein frame is computed from the kinetic terms of the (\hat{z}_1, \hat{z}_2) fields derived from (168). After putting $z_2 = 0$ at the zero mode level, the result is

$$V = \int dx_5 \sqrt{g_{55}} g^{55} \mathcal{K}^{a\bar{b}} \partial_5 z_a \partial_5 z_{\bar{b}} = \frac{4M_0^2}{(T + T^\dagger)^3} \frac{|1 - S|^2}{S + S^\dagger}, \quad (172)$$

where $a, b = 1, 2$ and $\mathcal{K}^{a\bar{b}}$ is the inverse of the Kähler metric $\mathcal{K}_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K}$. This result is interpreted as a superpotential generated for S . The 4d theory is completely described by

$$\mathcal{K} = -\ln(S + S^\dagger) - 3\ln(T + T^\dagger) \quad , \quad W = 2M_0(1 + S). \quad (173)$$

Notice that the superpotential corresponds to a non-perturbative effect from the heterotic viewpoint. The minimum of the scalar potential is $S = 1$ and corresponds to a spontaneously broken supergravity with zero cosmological constant. The order parameter for supersymmetry breaking is the gravitino mass $m_{3/2}^2 = e^{\mathcal{K}} |W|^2 = 2M_0^2 / (T + T^\dagger)^3$. If in 4d supergravity units we define $M_0 = \omega M_P$, then $m_{3/2} = \omega / R_5$, where R_5 is the radius of the fifth coordinate. This is consistent with the fact that the gravitino mass was affected by the R-symmetry. The Goldstone fermion is the fifth component of the $(Z_2^{HW} \text{ even})$ 5d gravitino Ψ_5 . The important point about (173) or any other supergravity example is that the breaking of supersymmetry à la Scherk-Schwarz appears to be spontaneous, of the F-type, with a zero cosmological constant. The resulting models are of no-scale type [111].

In heterotic strings the only available perturbative method of breaking supersymmetry is the Scherk-Schwarz mechanism. In this case, soft masses $M_{SUSY} \sim R^{-1}$ are generated at tree-level for the gauginos, so that phenomenologically interesting values require $R^{-1} \sim \text{TeV}$. In this case however the model cannot be controlled [81], in view of the large value of the heterotic string scale $M_H \sim 5 \times 10^{17} \text{ GeV}$. The most popular mechanism invoked in this case for breaking supersymmetry is gaugino condensation in a hidden sector [96] $\langle \lambda\lambda \rangle = \Lambda^3$, while the transmission to the observable sector is mediated by gravitational interactions, and thus

$$M_{SUSY} \sim \frac{\Lambda^3}{M_P^2}, \quad \text{where} \quad \Lambda \sim M_P e^{-1/(2b_0 g^2(M_P))}. \quad (174)$$

This mechanism singles out intermediate scales $\Lambda \sim 10^{12} - 10^{13} \text{ GeV}$, naturally realized by the one-loop running of the hidden sector gauge coupling and could also be useful for purposes like neutrino masses or PQ axions. Gaugino condensation, however, is a nonperturbative field theory phenomenon and there is little hope to discover a string theory description of it. A third possibility is to start directly with nonsupersymmetric strings, possibly interpreted as models with supersymmetry broken at the string scale $M_{SUSY} \sim M_H$. As M_H is very large, however, this possibility was completely ignored since there was no clear way to solve the hierarchy problem in this case.

In models with D-branes there are many different ways in which supersymmetry can be broken in a phenomenologically interesting way. This is due to the two main new features of these theories:

- The Standard Model can be confined to a subspace (D-brane) of the full ten or eleven dimensional space.
- The string scale in these models can be lowered all the way down to the TeV range.
- **Perturbative supersymmetry breaking with branes**

The simplest string constructions of this type were presented in Section 7. Even if several

distinct mechanisms are available, they can be splitted for phenomenological purposes into two classes :

- i) Supersymmetry broken in the bulk.
- ii) Supersymmetry broken on some branes.

The class i) contains the models with breaking through compactification, in which generically a one-loop cosmological constant is generated in the closed sector $E_0 \sim R^{-4}$, where R is the radius of the compact dimension Y breaking supersymmetry à la Scherk-Schwarz. If the Standard Model lives on a brane parallel to Y , then $M_{SUSY} \sim R^{-1}$ and the phenomenology is very close to the analogous heterotic models [81]. On the other hand, if the Standard Model lives on a brane perpendicular to Y , then at tree-level the massless brane spectrum is supersymmetric [19] and supersymmetry breaking on the brane is transmitted via radiative corrections. If the bulk (Y in this case) contains only gravity, then $M_{SUSY} \sim R^{-2}/M_P$ and we need here some intermediate radius $R^{-1} \sim 10^{11}$ GeV, natural in M-theory [34, 32] or intermediate scale string scenarios [112]. If the bulk contains also some other branes, there is also a Standard Model gauge mediation coming from states charged under both Standard Model and the bulk gauge groups [97, 112]. If the bulk volume is large, the bulk gauge coupling is volume suppressed with respect to the Standard Model couplings. Consequently, in this case also $M_{SUSY} \ll R^{-1}$ and the gauge transmission has all the known advantages concerning the flavor-blind structure of the resulting soft breaking terms (for a review and extensive references, see [98]).

Class ii) contains the “Brane supersymmetry breaking” models, with branes D and antibranes \bar{D} and a tree-level supersymmetric bulk spectrum. Supersymmetry is broken on the antibranes at the string scale M_I , while the tree-level spectrum of the branes is supersymmetric. If the Standard Model lives on the antibranes, the string scale M_I should be in the TeV range. Supersymmetry breaking on the branes can be transmitted, as before, by gravitational interactions, in which case $M_{SUSY} \sim M_I^2/M_P$ and, if the Standard Model

lives on the branes, phenomenology asks for $M_I \sim 10^{11}$ GeV. Alternatively, if the bulk volume is sufficiently small, the transmission can be gauge mediated and proceed through the massive brane-antibrane excitations. As discussed at the end of Section 3 and in Section 7, these models have uncanceled NS-NS tadpoles that translate, in physical terms, into scalar potentials for the dilaton and the moduli fields describing the compact space. In order to understand qualitatively some of their features, we briefly discuss a 6d model based on a toroidal compactification with D9 and $\bar{\text{D}}9$ branes with Chan-Paton factors N_+ , N_- and D5 and $\bar{\text{D}}5$ branes with Chan-Paton factors D_+ , D_- , worked out in [25]. The RR tadpole conditions read

$$N_+ - N_- = 32 \quad , \quad D_+ - D_- = 0 \quad , \quad (175)$$

and the scalar potential induced by the NS-NS tadpoles is

$$V_{eff} \sim e^{-\phi_6} \left[(N_+ + N_- - 32)\sqrt{v} + \frac{(D_+ + D_-)}{\sqrt{v}} \right] \quad (176)$$

in string units, where Φ_6 is the 6d dilaton. The potential has a minimum and stabilizes the internal space at the value $v_0 = (D_+ + D_-)/(N_+ + N_- - 32)$. We see that, in order to have a very large (very small) compact space, we need a very large number of D9+ $\bar{\text{D}}9$ (D5+ $\bar{\text{D}}5$) branes, compatible with (175). This is in principle possible, but of course asks for a dynamical explanation of the very large number of branes and antibranes in the model²⁰. The dilaton potential, on the other hand, has a runaway behavior. This is an unavoidable consequence of the perturbative nature of this mechanism. This problem is related to the dilaton tadpole, which asks for a redefinition of the spacetime background. The class ii) also contains models with supersymmetry breaking induced by internal magnetic fields.

- Nonperturbative supersymmetry breaking

²⁰The possible existence of a large number of (anti)branes in these models is a nontrivial and interesting possibility, since in supersymmetric compactifications the number of D9 or D5 branes of a given type is always equal or less than 32.

Despite of the serious progress achieved in the perturbative breaking of supersymmetry in Type I strings, we probably need nonperturbative effects for at least one reason mentioned above, the dilaton stabilization problem. Indeed, even if the problem can be circumvented searching for a nontrivial background à la Fischler-Susskind, the explicit example worked out in [58] suggests that nonperturbative effects in the string coupling (dilaton) appear in this background. The present-day technology forces us to rely here on field theoretical arguments, like holomorphy in supersymmetry. There are here several scenarios proposed in the literature, which take advantage of the various brane configurations, each brane different and far away from ours being a potential hidden sector breaking supersymmetry. A novelty in Type I is that twisted fields M_k can easily participate to supersymmetry breaking. A simple example is provided by gaugino condensation (162) with a large class of Kähler potentials for twisted moduli fields, and in particular the minimal one, $M_k^\dagger M_k$.

11. Bulk physics: Neutrino and axion masses with large extra dimensions

There is more and more convincing evidence for the existence of neutrino masses and mixings, in light of the recent SuperKamiokande results [113]. Any extension of the Standard Model should therefore address this question, at least at a qualitative level. The most elegant mechanism for explaining the smallness of neutrino masses postulates the existence of right-handed neutrinos with very large Majorana masses $10^{11} \text{ GeV} \leq M \leq 10^{15} \text{ GeV}$. Via the seesaw mechanism [99] very small neutrino masses, of the order of $m_\nu \sim v^2/M$, are generated, where $v \simeq 246 \text{ GeV}$ is the vev of the Higgs field. This suggests the presence of a large (intermediate or GUT) scale in the theory, related to new physics. On the other hand, low-scale string models do not have such a large scale and therefore superficially have problems to accommodate neutrino masses. Similarly, the strong CP problem in the Stan-

standard Model finds its most natural explanation by postulating a global continuous $U(1)_{PQ}$ symmetry with $U(1)_{PQ}[SU(3)]^2$ anomalies. In this case, the θ parameter of QCD becomes a dynamical field $\theta \rightarrow \theta + (1/f)a$, where a is called the Peccei-Quinn *axion* [114]. The symmetry $U(1)_{PQ}$ is spontaneously broken at a large scale f and, by instanton effects, an axion potential is generated such that $\theta + (1/f) \langle a \rangle = 0$, dynamically solving the strong CP problem. The experimentally allowed window for the axion is considered to be $10^8 \text{ GeV} \leq f \leq 10^{12} \text{ GeV}$. These arguments were used in [112] for arguing that the string scale is likely to be at some intermediate value $M_I \sim 10^{11} \text{ GeV}$.

In this Section it will be argued that there is actually a natural way to find very small neutrino and invisible axion masses, taking advantage of the fact that right-handed neutrinos and axions, that are Standard Model gauge singlets, can be placed in the bulk space. These scenarios have interesting new features compared to the standard 4d mechanisms due to the higher-dimensional nature of the gauge singlets.

- Neutrino masses

The scenario we present here is based on the observation that right-handed neutrinos can be put in the bulk of a very large (mm size) compact space [116, 117, 118], perpendicular to the brane where we live. Consider for simplicity the case of one family of neutrinos. The model consists of our brane with the left-handed neutrino ν_L and Higgs field confined to it and one bulk Dirac neutrino, $\Psi = (\psi_1, \bar{\psi}_2)^T$ in Weyl notation, invading a space with (again for simplicity) one compact perpendicular direction y . The compact direction is taken here to be an orbifold S^1/Z_2 , since as is well known circle compactifications are not phenomenologically realistic. The Z_2 orbifold acts on the spinors as $Z_2\Psi(y) = \pm\gamma_5\Psi(-y)$, so that one of the two-component Weyl spinors, *e.g.*, ψ_1 , is even under the Z_2 action $y \rightarrow -y$, while the other spinor ψ_2 is odd. If the left-handed neutrino ν_L is restricted to a brane located at the orbifold fixed point $y = 0$, ψ_2 vanishes at this point and so ν_L couples

only to ψ_1 . This then results in a Lagrangian of the form

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \int d^4x dy M_s \left\{ \bar{\psi} i \bar{\gamma}^M \partial_M \psi - \partial_M \bar{\psi} i \bar{\gamma}^M \psi \right\} \\ & - \int d^4x \left\{ \bar{\nu}_L i \bar{\sigma}^\mu D_\mu \nu_L + (\hat{m} \nu_L \psi_1|_{y=0} + \text{h.c.}) \right\} . \end{aligned} \quad (177)$$

Here M_s is the mass scale of the higher-dimensional fundamental theory (*e.g.*, a reduced Type I string scale) and the spacetime index M runs over all five dimensions: $x^M \equiv (x^\mu, y)$. The first line describes the kinetic-energy term for the 5d Ψ field, while the second line describes the kinetic energy of the 4d two-component neutrino field ν_L , as well as the coupling between ν_L and ψ_1 . Note that in 5d, a bare Dirac mass term for Ψ would not have been invariant under the action of the Z_2 orbifold, since $\bar{\Psi}\Psi \sim \psi_1\psi_2 + \text{h.c.}$

Now compactify the Lagrangian (177) down to 4d, expanding the 5d Ψ field in Kaluza-Klein modes. The orbifold relations $\psi_{1,2}(-y) = \pm \psi_{1,2}(y)$ imply that the Kaluza-Klein decomposition takes the form

$$\psi_1(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi_1^{(n)}(x) \cos(ny/R) , \quad \psi_2(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \psi_2^{(n)}(x) \sin(ny/R) . \quad (178)$$

However, a more general possibility emerges naturally from the Scherk-Schwarz compactification [79]. Recall that our original 5d Dirac spinor field Ψ is decomposed in the Weyl basis as $\Psi = (\psi_1, \bar{\psi}_2)^T$, where ψ_1 and ψ_2 have the mode expansions given in (178). Let us consider performing a local rotation in (ψ_1, ψ_2) space of the form

$$\begin{pmatrix} \hat{\psi}_1 \\ \hat{\psi}_2 \end{pmatrix} \equiv U \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \text{where} \quad U \equiv \begin{pmatrix} \cos(\omega y/R) & -\sin(\omega y/R) \\ \sin(\omega y/R) & \cos(\omega y/R) \end{pmatrix} , \quad (179)$$

with ω an (for the moment) arbitrary real number. The effect of the matrix U in (179) is to twist the fermions after a $2\pi R$ rotation on y . Such twisted boundary conditions, as we have seen, are allowed in field and in string theory if the higher-dimensional theory has a suitable $U(1)$ symmetry. The 4d Lagrangian of the component fields coming from the 5d Lagrangian is found from (177) by replacing everywhere $\psi_i \rightarrow \hat{\psi}_i$, and includes the mass

terms

$$\begin{aligned} \mathcal{L}_{kin} = & -\frac{1}{2} \int dy \, M_s \left\{ \bar{\psi} i \gamma^5 \partial_5 \hat{\psi} - \partial_5 \bar{\psi} i \gamma^5 \hat{\psi} \right\} = \\ & - \sum_{n=0}^{\infty} \left\{ \frac{n}{R} \psi_1^{(n)} \psi_2^{(n)} + \frac{M_0}{2} (\psi_1^{(n)} \psi_1^{(n)} + \psi_2^{(n)} \psi_2^{(n)}) + h.c. \right\} , \end{aligned} \quad (180)$$

where $M_0 = \omega/R$. For convenience, let us define the linear combinations $N^{(n)} \equiv (\psi_1^{(n)} + \psi_2^{(n)})/\sqrt{2}$ and $M^{(n)} \equiv (\psi_1^{(n)} - \psi_2^{(n)})/\sqrt{2}$ for all $n > 0$.

Inserting (179), (180) into (177) and integrating over the compactified dimension then yields

$$\begin{aligned} \mathcal{L} = & - \int d^4x \left\{ \bar{\nu}_L i \bar{\sigma}^\mu D_\mu \nu_L + \bar{\psi}_1^{(0)} i \bar{\sigma}^\mu \partial_\mu \psi_1^{(0)} + \sum_{n=1}^{\infty} \left(\bar{N}^{(n)} i \bar{\sigma}^\mu \partial_\mu N^{(n)} + \bar{M}^{(n)} i \bar{\sigma}^\mu \partial_\mu M^{(n)} \right) \right. \\ & + \left\{ \frac{1}{2} M_0 \psi_1^{(0)} \psi_1^{(0)} + \frac{1}{2} \sum_{n=1}^{\infty} \left[\left(M_0 + \frac{n}{R} \right) N^{(n)} N^{(n)} + \left(M_0 - \frac{n}{R} \right) M^{(n)} M^{(n)} \right] \right. \\ & \left. \left. + m \left[\nu_L \psi_1^{(0)} + \nu_L \sum_{n=1}^{\infty} \left(N^{(n)} + M^{(n)} \right) \right] + h.c. \right\} \right\} . \end{aligned} \quad (181)$$

Here the first line gives the four-dimensional kinetic-energy terms, while the second line gives the Kaluza-Klein and Majorana mass terms. The third line of (181) describes the coupling between the 4d neutrino ν_L and the 5d field Ψ . Note that in obtaining this Lagrangian it is necessary to rescale the Kaluza-Klein modes $\psi_1^{(0)}$, $N^{(n)}$, and $M^{(n)}$ so that their 4d kinetic-energy terms are canonically normalized. This then results in a suppression of the Dirac neutrino mass \hat{m} by the factor $(2\pi M_s R)^{1/2}$. In the third line, we have therefore defined the effective Dirac neutrino mass couplings

$$m \equiv \frac{\hat{m}}{\sqrt{2} \sqrt{\pi M_s R}} . \quad (182)$$

In the Lagrangian (181), the Standard-Model neutrino ν_L mixes with the entire tower of Kaluza-Klein states of the higher-dimensional Ψ field. Indeed, if for simplicity we restrict our attention to the case of only one extra dimension and define

$$\mathcal{N}^T \equiv (\nu_L, \psi_1^{(0)}, N^{(1)}, M^{(1)}, N^{(2)}, M^{(2)}, \dots) , \quad (183)$$

we see that the mass terms in the Lagrangian (181) take the form $(1/2)(\mathcal{N}^T \mathcal{M} \mathcal{N} + \text{h.c.})$, where the mass matrix is

$$\mathcal{M} = \begin{pmatrix} 0 & m & m & m & m & m & \dots \\ m & M_0 & 0 & 0 & 0 & 0 & \dots \\ m & 0 & M_0 + 1/R & 0 & 0 & 0 & \dots \\ m & 0 & 0 & M_0 - 1/R & 0 & 0 & \dots \\ m & 0 & 0 & 0 & M_0 + 2/R & 0 & \dots \\ m & 0 & 0 & 0 & 0 & M_0 - 2/R & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \quad (184)$$

Let us start for simplicity by disregarding the possible bare Majorana mass term, setting $M_0 = 0$. In this case, the characteristic polynomial which determines the eigenvalues λ of the mass matrix (184) can be worked out exactly and takes the form

$$\left[\prod_{k=1}^{\infty} \left(\frac{k^2}{R^2} - \lambda^2 \right) \right] \left[\lambda^2 - m^2 + 2\lambda^2 m^2 R^2 \sum_{k=1}^{\infty} \frac{1}{k^2 - \lambda^2 R^2} \right] = 0 , \quad (185)$$

clearly invariant under $\lambda \rightarrow -\lambda$. From this we immediately see that all eigenvalues fall into *degenerate*, pairs of opposite sign. In order to solve this eigenvalue equation, it is convenient to note that $\lambda = k/R$ is never a solution (unless of course $m = 0$), as the cancellation that would occur in the first factor in (185) is offset by the divergence of the second factor. We are therefore free to disregard the first factor entirely, and focus on solutions for which the second factor vanishes. The summation in second factor can be performed exactly, resulting in the transcendental equation

$$\lambda R = \pi(mR)^2 \cot(\pi\lambda R) . \quad (186)$$

All the eigenvalues can be determined from this equation, as functions of the product mR . This equation can be analyzed graphically [116], and in the limit $mR \rightarrow 0$ (corresponding to $m \rightarrow 0$), the eigenvalues are k/R , $k \in \mathbb{Z}$, with a double eigenvalue at $k = 0$. Conversely, in the limit $mR \rightarrow \infty$, the eigenvalues with $k > 0$ shift smoothly toward $(k + 1/2)/R$, while

those with $k < 0$ shift smoothly toward $(k - 1/2)/R$. Finally, the double zero eigenvalue splits toward the values $\pm 1/(2R)$. In order to derive general analytical expressions valid in the limit $mR \ll 1$, we can solve (186) iteratively by power-expanding the cotangent function. To order $\mathcal{O}(m^5 R^5)$, this gives the solutions

$$\lambda_{\pm} = \pm m \left(1 - \frac{\pi^2}{6} m^2 R^2 + \dots \right), \quad \lambda_{\pm k} = \pm \frac{k}{R} \left(1 + \frac{m^2 R^2}{k^2} - \frac{m^4 R^4}{k^4} + \dots \right), \quad (187)$$

where $\lambda_{\pm k}$ are the two eigenvalues at each Kaluza-Klein level k and λ_{\pm} are the “light” eigenvalues at $k = 0$. Finally, it is also straightforward to solve explicitly for the light mass eigenstates $|\tilde{\nu}_{\pm}\rangle$ corresponding to $k = 0$. To leading order in mR , we find

$$|\tilde{\nu}_{\pm}\rangle = \frac{1}{\sqrt{2}} \left\{ \left(1 - \frac{\pi^2}{6} m^2 R^2 \right) |\nu_L\rangle \pm |\psi_1^{(0)}\rangle - mR \sum_{k=1}^{\infty} \frac{1}{k} \left[|N^{(k)}\rangle - |M^{(k)}\rangle \right] \right\}. \quad (188)$$

This implies that the overlap between the light mass eigenstates and the neutrino gauge eigenstate is generically less than half in this scenario. The important prediction of this scenario is that the gauge neutrino and the (lightest) sterile neutrino are degenerate in mass, a possibility that can be experimentally tested.

Let us now return to the more general case $M_0 \neq 0$. To this end, it is useful to define

$$k_0 \equiv [M_0 R], \quad \epsilon \equiv M_0 - \frac{k_0}{R}, \quad (189)$$

where $[x]$ denotes here the integer nearest to x . Thus, ϵ is the smallest diagonal entry in the mass matrix (184), corresponding to the excited Kaluza-Klein state $M^{(k_0)}$. In other words, $\epsilon \equiv M_0$ (modulo R^{-1}) satisfies $-1/(2R) < \epsilon \leq 1/(2R)$. The remaining diagonal entries in the mass matrix can then be expressed as $\epsilon \pm k'/R$, where $k' \in Z^+$. Reordering

the rows and columns of our mass matrix, we can therefore cast it into the form

$$\mathcal{M} = \begin{pmatrix} 0 & m & m & m & m & m & \dots \\ m & \epsilon & 0 & 0 & 0 & 0 & \dots \\ m & 0 & \epsilon + 1/R & 0 & 0 & 0 & \dots \\ m & 0 & 0 & \epsilon - 1/R & 0 & 0 & \dots \\ m & 0 & 0 & 0 & \epsilon + 2/R & 0 & \dots \\ m & 0 & 0 & 0 & 0 & \epsilon - 2/R & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (190)$$

While this is of course nothing but the original mass matrix (184), the important consequence of this rearrangement is that the *heavy* mass scale M_0 has been replaced by the *light* mass scale ϵ . Unlike M_0 , we see that $|\epsilon| \sim \mathcal{O}(R^{-1})$. Thus, the heavy Majorana mass scale M_0 completely *decouples* from the physics. Indeed, the value of M_0 enters the results only through its determinations of k_0 and the precise value of ϵ . Therefore, interestingly enough, the presence of the infinite tower of regularly-spaced Kaluza-Klein states ensures that only the value of M_0 modulo R^{-1} plays a role.

The easiest way to solve (190) for the eigenvalues λ_{\pm} is to integrate out the Kaluza-Klein modes. It turns out that there are two relevant cases to consider, depending on the value of ϵ . If $|\epsilon| \gg m$ (which can arise when $mR \ll 1$), *all* of the Kaluza-Klein modes are extremely massive relative to m , and we can integrate them out to obtain an effective $\nu_L \nu_L$ mass term of size

$$\begin{aligned} |\epsilon| \gg m : \quad m_{\nu} &= m^2/\epsilon + m^2 \sum_{k'=1}^{\infty} \left(\frac{1}{\epsilon + k'/R} + \frac{1}{\epsilon - k'/R} \right) \\ &= \pi m^2 R \cot(\pi R \epsilon) . \end{aligned} \quad (191)$$

We shall discuss the special case $\epsilon = 1/2R$ later on. Alternatively, if $|\epsilon| \not\gg m$, the lightest Kaluza-Klein mode $M^{(k_0)}$ should not be integrated out, and the end result is an effective $\nu_L \nu_L$ mass term of size μ , where

$$|\epsilon| \not\gg m : \quad \mu \equiv -m^2 \sum_{k'=1}^{\infty} \left(\frac{1}{\epsilon + k'/R} + \frac{1}{\epsilon - k'/R} \right)$$

$$= \frac{m^2}{\epsilon} - \pi m^2 R \cot(\pi R \epsilon) . \quad (192)$$

Note that $\mu \rightarrow 0$ smoothly as $\epsilon \rightarrow 0$, with μ otherwise of size $\mathcal{O}(m^2 R)$. Diagonalizing the final 2×2 mass matrix mixing ν_L and $M^{(k_0)}$ in the presence of this mass term then yields

$$|\epsilon| \not\gg m : \quad \lambda_{\pm} = \frac{1}{2} \left[(\mu + \epsilon) \pm \sqrt{(\mu - \epsilon)^2 + 4m^2} \right] . \quad (193)$$

Thus, as $M_0 \rightarrow 0$ (or as $M_0 \rightarrow n/R$ where $n \in \mathbb{Z}$), we see that $\epsilon, \mu \rightarrow 0$, and we recover the eigenvalues given in (187).

We therefore conclude that, although we may have started with a bare Majorana mass $M_0 \gg R^{-1}$, in all cases the final neutrino mass remains of order $m^2 R$. Even though we might have expected a neutrino mass of order m^2/M_0 from the mixing between ν_L and the original zero-mode $\psi_1^{(0)}$, the contribution m^2/M_0 from the zero-mode is completely canceled by the summation over the Kaluza-Klein tower, while the seesaw between ν_L and $M^{(k_0)}$ becomes dominant. It is this feature that causes the heavy scale M_0 to be effectively replaced by the radius R^{-1} , so that once again our effective seesaw scale is $M_{\text{eff}} \sim \mathcal{O}(R^{-1})$.

In string theory, however, there are additional topological constraints (coming from the preservation of the form of the worldsheet supercurrent) that permit only *discrete* values of ω [78]. In particular, in a compactification from five to four dimensions, this restriction allows only one non-trivial possibility, $\omega = 1/2$. Taking $\omega = 1/2$ then implies $\psi_{1,2}(2\pi R) = -\psi_{1,2}(0)$, which shows that lepton number is broken globally (although not locally) as the spinor is taken around the compactified space. In order to obtain the corresponding neutrino mass, we note that for $\epsilon = 1/2R$ the assumption $mR \ll 1$ translates into $\epsilon \gg m$, whereupon the result (191) is valid. Thus, for $\epsilon = 1/2R$ we find the remarkable result that $m_{\nu} = 0$! In obtaining this result, one might worry that (191) is only approximate because it relies on the procedure of integrating out the Kaluza-Klein states rather than on a full diagonalization of the corresponding mass matrix. However, it is straightforward to show that when $\epsilon = 1/2R$, the characteristic eigenvalue equation $\det(\mathcal{M} - \lambda I) = 0$ for

the mass matrix (184),(190) becomes

$$\lambda R \left[\prod_{k=1}^{\infty} (\lambda^2 R^2 - (k - \frac{1}{2})^2) \right] \left[1 - 2m^2 R^2 \sum_{k=1}^{\infty} \frac{1}{\lambda^2 R^2 - (k - 1/2)^2} \right] = 0 . \quad (194)$$

This has an exact trivial solution $\lambda = 0$, corresponding to an exactly massless neutrino. Indeed, the characteristic polynomial for the mass matrix in this case has the form

$$\lambda R = -\pi(mR)^2 \tan(\pi\lambda R) . \quad (195)$$

It is then clear than the zero eigenvalue is always present, irrespective of the value of the radius. In fact, by changing the value of M_0 , we see that it is possible to smoothly *interpolate* between the scenario with $M_0 = 0$ and the scenario we are discussing here [116]. This also provides another explanation of why only the value $\epsilon \sim M_0$ (modulo R^{-1}) is relevant physically. The regular, repeating aspect of the infinite towers of Kaluza-Klein states is now manifested graphically in the periodic nature of the cotangent function.

We can also solve for the full spectrum of eigenvalues as a function of mR . We find that the non-zero eigenvalues are identical to those given in (187) for $k \neq 0$, but now $k \rightarrow k - 1/2$. Note that the massless neutrino eigenstate is *primarily* composed of the neutrino gauge eigenstate ν_L , for $mR \ll 1$. Although this neutrino mass eigenstate also contains a small, non-trivial admixture of Kaluza-Klein states, its dominant component is still the gauge-eigenstate neutrino ν_L , as required phenomenologically. It should be stressed that this combined neutrino mass eigenstate is exactly massless in the limit that the full, infinite tower of Kaluza-Klein states participates in the mixing²¹. This result is valid *regardless* of the value of neutrino Yukawa coupling m or of the scale R^{-1} of the Kaluza-Klein states.

²¹Actually, our field theory approach breaks down for KK masses of the order of the fundamental string scale M_s . If we cut our summation at $k_{max} = RM_s$, the physical neutrino is not exactly massless anymore, but acquires a small mass $m_\nu \sim m^2/M_s$. For phenomenologically interesting values $m \sim R^{-1} \simeq 10^{-2}eV$ and $M_s \sim \text{TeV}$, this mass is however negligibly small $m_\nu \sim 10^{-15}eV$.

It is also interesting to notice that the desired value of the Majorana mass $M_0 = 1/2R$ emerges naturally from a Scherk-Schwarz decomposition, for reasons that are *topological* and hence do not require any fine-tuning. It should however be stressed that in this case lepton number is not broken if we consider the full tower of KK states. Indeed, it can be easily shown that for $\omega = 1/2$ KK states pair up so that the full lagrangian still preserves lepton number.

The scenario(s) presented have also other interesting consequences. The neutrino eigenstate can now oscillate into an infinite tower of right-handed KK neutrinos with a probability that can be reliably estimated and experimentally tested. Moreover, even if in the last scenario presented the physical neutrino is massless, its probability of oscillation into the tower of KK states is nonvanishing. In particular, a neutrino mass difference $\Delta m \sim 10^{-2}eV$, that fits the experimental data, could well be explained by an oscillation of the massless neutrino into the first KK state, for a radius $R^{-1} \sim 10^{-2}eV$, precisely in the mm region we are interested in !

- Bulk axion masses

The most elegant explanation of the strong CP problem is provided by the Peccei-Quinn (PQ) mechanism [114], in which the CP violating angle $\bar{\Theta}$ ($\bar{\Theta}$ by definition includes the contribution of weak interactions) is set to zero dynamically as a result of a global, spontaneously broken $U(1)_{PQ}$ Peccei-Quinn symmetry. However, associated with this symmetry there is a new Nambu-Goldstone boson, the axion [115], which essentially replaces the $\bar{\Theta}$ parameter in the effective Lagrangian. This then results in an effective Lagrangian of the form

$$\mathcal{L}^{\text{eff}} = \mathcal{L}_{\text{QCD}} - \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_{\text{PQ}}} \xi \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{\mu\nu a} \ , \quad (196)$$

where f_{PQ} is the axion decay constant, associated with the scale of PQ symmetry breaking. Here ξ is a model-dependent parameter describing the PQ transformation properties of the ordinary fermions, and we have not exhibited other terms in the Lagrangian that describe

axion/fermion couplings. The mass of the axion is then expected to be of the order

$$m_a \sim \frac{\Lambda_{\text{QCD}}^2}{f_{\text{PQ}}} , \quad (197)$$

where $\Lambda_{\text{QCD}} \approx 250$ MeV; likewise, the couplings of axions to fermions are suppressed by a factor of $1/f_{\text{PQ}}$. Thus, heavier scales for PQ symmetry breaking generally imply lighter axions that couple more weakly to ordinary matter.

Ordinarily, one might have preferred to link the scale f_{PQ} to the scale of electroweak symmetry breaking, thus implying an axion mass $m_a \approx \mathcal{O}(10^2)$ keV. However, so far all experimental searches for such axions have been unsuccessful [119], and indeed only a narrow allowed window exists:

$$10^{10} \text{ GeV} \leq f_{\text{PQ}} \leq 10^{12} \text{ GeV} , \quad 10^{-5} \text{ eV} \leq m_a \leq 10^{-3} \text{ eV} . \quad (198)$$

The resulting axion is exceedingly light and its couplings to ordinary matter are exceedingly suppressed. These bounds generally result from various combinations of laboratory, astrophysical, and cosmological constraints. In all cases, however, the crucial ingredient is the correlation between the *mass* of the axion and the strength of its *couplings* to matter, since both are essentially determined by the single parameter f_{PQ} .

This situation may be drastically altered in theories with large extra spatial dimensions. We shall consider the consequences of placing the PQ axion in the “bulk” (*i.e.*, perpendicular to the brane that contains the Standard Model) so that it accrues an infinite tower of Kaluza-Klein excitations [36, 121, 122]. This is reminiscent of the option of placing the right-handed neutrino in the bulk discussed above. In order to generalize the PQ mechanism, we will assume that there exists a complex scalar field ϕ in higher dimensions which transforms under a global $U(1)_{\text{PQ}}$ symmetry $\phi \rightarrow e^{i\Lambda}\phi$. This symmetry is assumed to be spontaneously broken by the bulk dynamics so that $\langle\phi\rangle = f_{\text{PQ}}/\sqrt{2}$, where f_{PQ} is the energy scale associated with the breaking of the PQ symmetry. We thus write our complex

scalar field ϕ in the form

$$\phi \approx \frac{f_{\text{PQ}}}{\sqrt{2}} e^{ia/f_{\text{PQ}}} , \quad (199)$$

where a is the Nambu-Goldstone boson (axion) field. If we concentrate on the case of 5d for concreteness, the kinetic-energy term for the scalar field takes the form

$$\mathcal{S}_{\text{K.E.}} = - \int d^4x dy M_s \partial_M \phi^* \partial^M \phi = - \int d^4x dy M_s \frac{1}{2} \partial_M a \partial^M a , \quad (200)$$

where we have neglected the contributions of the radial mode. Here x^μ are the 4d coordinates and y is the coordinate of the fifth dimension. Note that there is no mass term for the axion, as this would not be invariant under the $U(1)_{\text{PQ}}$ transformation $a \rightarrow a + f_{\text{PQ}} \Lambda$. Furthermore, as a result of the chiral anomaly, we will also assume a bulk/boundary coupling of the form

$$\mathcal{S}_{\text{coupling}} = \int d^4x dy \frac{\xi}{f_{\text{PQ}}} \frac{g^2}{32\pi^2} a F_a^{\mu\nu} \tilde{F}_{\mu\nu a} \delta(y) , \quad (201)$$

where $F_{\mu\nu a}$ is the (4d) QCD field strength confined to a four-dimensional subspace (*e.g.*, a D-brane) located at $y = 0$. Thus, our effective 5d action takes the form

$$\mathcal{S}_{\text{eff}} = \int d^4x dy \left[-\frac{1}{2} M_s \partial_M a \partial^M a + \frac{\xi}{f_{\text{PQ}}} \frac{g^2}{32\pi^2} a F_a^{\mu\nu} \tilde{F}_{\mu\nu a} \delta(y) \right] . \quad (202)$$

While we have assumed that the spontaneously broken $U(1)_{\text{PQ}}$ is parametrized by f_{PQ} , one still has to address the fact that gravitational effects can also break the $U(1)_{\text{PQ}}$ symmetry, since gravitational interactions generically break global symmetries [120]. We will assume, however, that the gravitational contributions to the axion mass are indeed suppressed, and that $U(1)_{\text{PQ}}$ remains a valid symmetry even in the presence of gravitational effects.

In order to obtain an effective 4d theory, our next step is to compactify the fifth dimension. For simplicity, we shall assume that this dimension is compactified on the Z_2 orbifold that we considered in the neutrino case. This implies that the axion field will have a Kaluza-Klein decomposition of the form

$$a(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} a_n(x^\mu) \cos\left(\frac{ny}{R}\right) , \quad (203)$$

where $a_n(x^\mu) \in R$ are the Kaluza-Klein modes and where we have demanded that the axion field be symmetric under the Z_2 action (in order to have a zero-mode that we can identify with the usual 4d axion).

It is also interesting to note that for the Kaluza-Klein axion modes a_n , the Peccei-Quinn transformation takes the form

$$\begin{cases} a_0 \rightarrow a_0 + f_{\text{PQ}}\Lambda \\ a_k \rightarrow a_k \end{cases} \quad \text{for all } k > 0 . \quad (204)$$

Thus, only a_0 serves as the true axion transforming under the PQ transformation, while the excited Kaluza-Klein modes a_k remain invariant.

Substituting (203) into (202) and integrating over the fifth dimension, we obtain the effective four-dimensional Lagrangian density

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} - \frac{1}{2} \sum_{n=0}^{\infty} (\partial_\mu a_n)^2 - \frac{1}{2} \sum_{n=1}^{\infty} \frac{n^2}{R^2} a_n^2 + \frac{\xi}{\hat{f}_{\text{PQ}}} \frac{g^2}{32\pi^2} \left(\sum_{n=0}^{\infty} r_n a_n \right) F_a^{\mu\nu} \tilde{F}_{\mu\nu a} , \quad (205)$$

where

$$r_n \equiv \begin{cases} 1 & \text{if } n = 0 \\ \sqrt{2} & \text{if } n > 0 \end{cases} . \quad (206)$$

Note that in order to obtain (205) and (206), we had to rescale each of the Kaluza-Klein modes a_n in order to ensure that they have canonically normalized kinetic-energy terms. We have also defined $\hat{f}_{\text{PQ}} \equiv (VM_s)^{1/2} f_{\text{PQ}}$, where V is the volume of our compactified space. For δ extra dimensions, this definition generalizes to $\hat{f}_{\text{PQ}} \equiv (VM_s^\delta)^{1/2} f_{\text{PQ}}$. Note that while f_{PQ} sets the overall mass scale for the breaking of the Peccei-Quinn symmetry, it is the volume-renormalized quantity \hat{f}_{PQ} that parametrizes the coupling between the axion and the gluons. In general, since $M_s \gg R^{-1}$, we find that $\hat{f}_{\text{PQ}} \gg f_{\text{PQ}}$. Therefore, as pointed out in Ref. [36], this volume-renormalization of the brane/bulk coupling can be used to obtain sufficiently suppressed axion/gauge-field couplings even if f_{PQ} itself is taken to be relatively small. Notice that, if we were to take $\delta = n$ for the current axion case, (149) would imply either that $\hat{f}_{\text{PQ}} \sim M_{\text{Planck}}$ (which would presumably overclose the universe), or $M_s \ll \mathcal{O}(\text{TeV})$ (which would clearly violate current experimental bounds). Therefore,

if we assume an isotropic compactification with all equal radii, an intermediate scale \hat{f}_{PQ} can be generated only if $\delta < n$. In other words, the axion must be restricted to a *subspace* of the full higher-dimensional bulk.

Let us now proceed to verify that this higher-dimensional PQ mechanism still cancels the CP-violating phase, and use this to calculate the *mass* of the axion. In the one-instanton dilute-gas approximation, it is straightforward to show that

$$\langle F_a^{\mu\nu} \tilde{F}_{\mu\nu a} \rangle = -\Lambda_{\text{QCD}}^4 \sin \left(\frac{\xi}{\hat{f}_{\text{PQ}}} \sum_{n=0}^{\infty} r_n a_n + \bar{\Theta} \right) . \quad (207)$$

This gives rise to an effective potential for the axion modes:

$$V(a_n) = \frac{1}{2} \sum_{n=1}^{\infty} \frac{n^2}{R^2} a_n^2 + \frac{g^2}{32\pi^2} \Lambda_{\text{QCD}}^4 \left[1 - \cos \left(\frac{\xi}{\hat{f}_{\text{PQ}}} \sum_{n=0}^{\infty} r_n a_n + \bar{\Theta} \right) \right] . \quad (208)$$

In order to exhibit the PQ mechanism, we now minimize the axion effective potential,

$$\frac{\partial V}{\partial a_n} = \frac{n^2}{R^2} a_n + r_n \frac{\xi}{\hat{f}_{\text{PQ}}} \frac{g^2}{32\pi^2} \Lambda_{\text{QCD}}^4 \sin \left(\frac{\xi}{\hat{f}_{\text{PQ}}} \sum_{n=0}^{\infty} r_n a_n + \bar{\Theta} \right) = 0 , \quad (209)$$

obtaining the unique solution

$$\begin{aligned} \langle a_0 \rangle &= \frac{\hat{f}_{\text{PQ}}}{\xi} (-\bar{\Theta} + \ell\pi) , \quad \ell \in 2Z \\ \langle a_k \rangle &= 0 \quad \text{for all } k > 0 . \end{aligned} \quad (210)$$

Note that while any value $\ell \in Z$ provides an extremum of the potential, only the values $\ell \in 2Z$ provide the desired *minima*. Thus, this higher-dimensional Peccei-Quinn mechanism still solves the strong CP problem: a_0 is the usual Peccei-Quinn axion which solves the strong CP problem by itself by cancelling the $\bar{\Theta}$ angle, while all of the excited Kaluza-Klein axions a_k for $k > 0$ have vanishing VEVs. This makes sense, since only a_0 is a true massless Nambu-Goldstone field from the 4d perspective (see the PQ transformation properties (204)). However, these excited Kaluza-Klein axion states nevertheless have a drastic effect on the axion mass matrix. Indeed, the mass matrix derived from (208) is

$$\mathcal{M}_{nn'}^2 \equiv \frac{n^2}{R^2} \delta_{nn'} + \xi^2 \frac{g^2}{32\pi^2} \frac{\Lambda_{\text{QCD}}^4}{\hat{f}_{\text{PQ}}^2} r_n r_{n'} \cos \left(\frac{\xi}{\hat{f}_{\text{PQ}}} \sum_{n=0}^{\infty} r_n a_n + \bar{\Theta} \right) \Big|_{\langle a \rangle} , \quad (211)$$

and in the vicinity of the minimum (210) becomes

$$\mathcal{M}_{nn'}^2 = \frac{n^2}{R^2} \delta_{nn'} + \xi^2 \frac{g^2}{32\pi^2} \frac{\Lambda_{\text{QCD}}^4}{\hat{f}_{\text{PQ}}^2} r_n r_{n'} . \quad (212)$$

Let us now define

$$m_{\text{PQ}}^2 \equiv \xi^2 \frac{g^2}{32\pi^2} \frac{\Lambda_{\text{QCD}}^4}{\hat{f}_{\text{PQ}}^2} , \quad y \equiv \frac{1}{m_{\text{PQ}} R} , \quad (213)$$

so that m_{PQ} is the expected mass that the axion would ordinarily have acquired in four dimensions (depending on \hat{f}_{PQ} rather than f_{PQ} itself), and y is the ratio of the scale of the extra dimension and m_{PQ} . Our mass matrix then takes the form

$$\mathcal{M}_{nn'}^2 = m_{\text{PQ}}^2 \left(r_n r_{n'} + y^2 n^2 \delta_{nn'} \right) , \quad (214)$$

or equivalently

$$\mathcal{M}^2 = m_{\text{PQ}}^2 \begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \dots \\ \sqrt{2} & 2 + y^2 & 2 & 2 & \dots \\ \sqrt{2} & 2 & 2 + 4y^2 & 2 & \dots \\ \sqrt{2} & 2 & 2 & 2 + 9y^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \quad (215)$$

Note that the usual Peccei-Quinn case corresponds to the upper-left 1×1 matrix, leading to the expected result $\mathcal{M}^2 = m_{\text{PQ}}^2$. Thus, the additional rows and columns reflect the extra KK states, and their physical effect is to pull the lowest eigenvalue of this matrix away from m_{PQ}^2 .

Deriving the condition for the eigenvalues of this matrix is straightforward. Let us denote the eigenvalues of this matrix as λ^2 rather than λ because this is a (mass)² matrix. We then find that the eigenvalues are the solutions to the transcendental equation

$$\frac{\pi \tilde{\lambda}}{y} \cot \left(\frac{\pi \tilde{\lambda}}{y} \right) = \tilde{\lambda}^2 , \quad (216)$$

where we have defined the dimensionless eigenvalue $\tilde{\lambda} \equiv \lambda/m_{\text{PQ}}$. In terms of dimensionful

quantities, this transcendental equation takes the equivalent form²²

$$\pi R \lambda \cot(\pi R \lambda) = \frac{\lambda^2}{m_{\text{PQ}}^2} . \quad (217)$$

We can check that (217) makes sense as $R \rightarrow 0$. In this limit, the KK states become infinitely heavy and decouple, and we are left with the lightest eigenvalue $\lambda = m_{\text{PQ}}$. As R increases, the effect of the additional large dimension is felt through a reduction of this lowest eigenvalue and, as a result, the mass of the lightest axion decreases [122].

One important consequence, easy to check plotting (217), is that the lightest axion mass eigenvalue m_a is strictly bounded by the radius

$$m_a \leq \frac{1}{2} R^{-1} . \quad (218)$$

This result holds *regardless* of the value of m_{PQ} . Thus, in higher dimensions, when $m_{\text{PQ}} \geq 1/2R$, the size of the axion mass is set by the radius R and not by the Peccei-Quinn scale f_{PQ} . and therefore the Peccei-Quinn scale essentially *decouples* from the axion mass. Indeed, as long as $m_{\text{PQ}} \geq 1/2R$, we see that $m_a \leq 1/2R$ *regardless* of the specific values of m_{PQ} or Λ_{QCD} .

This observation has a number of interesting implications. First, an axion mass in the allowed range (198) is already achieved for R in the submillimeter range, independently of m_{PQ} . Second, surprisingly m_{PQ} can still be lowered or raised at will without violating the constraint (198), provided $m_{\text{PQ}} \geq 1/2R$. In other words, having already satisfied the axion *mass* constraints by appropriately choosing the value of R , we are now essentially free to tune m_{PQ} (or equivalently the fundamental Peccei-Quinn symmetry breaking scale f_{PQ})

²² Interestingly, this eigenvalue equation is identical to the one that emerges [116] (see eq.(186) of the previous paragraph) when the right-handed neutrino ν_R is placed in the bulk, with the mass scale m_{PQ} in the axion case corresponding to the Dirac coupling m in the neutrino case. Remarkably, this correspondence exists even though the axion and right-handed neutrino have different spins, and even though the mechanisms for mass generation are completely different in the two cases.

in such a way as to weaken the axion couplings to matter and make the axion sufficiently invisible. This may therefore provide a new method of obtaining an invisible axion.

12. Low-scale string predictions for accelerators

One of the main motivations for low-scale string theories comes from the possibility of testing them at future colliders. Indeed, virtual string (oscillator) states appear in all string amplitudes, and in particular in tree-level Veneziano-type amplitudes, and give deviations from the field-theory amplitudes for energies $E \leq M_I$. In addition, there are effects of gravitational Kaluza-Klein states [101] both via their direct production and via indirect (virtual) effects in various cross-sections. This paragraph is devoted to the direct evaluation in the Type I string of the relevant amplitudes, that were estimated in a field-theory context in various papers [101]. We will argue, using the results obtained in [127] (see also [128]), that the full string amplitudes contain some new features that are relevant for the future accelerator searches.

An important notion that appear in string computations is that of *form factor*. In our case we are interested in the form factor in the brane-brane-bulk vertex and we shall consider, for definiteness, bulk gravitons. Let us assume for the moment that the form factor can be described by the local lagrangian

$$\mathcal{L}_{int} = \int d^4x d^{\delta}y \, h_{\mu\nu}(x, \mathbf{y}) B(\mathbf{y}) T^{\mu\nu}(x) , \quad (219)$$

where $h_{\mu\nu}$ denotes the graviton fluctuations, $T^{\mu\nu}$ denotes the matter energy-momentum tensor and $B(\mathbf{y})$ (which could also contain derivatives and could even be a nonlocal function) describes the brane “thickness”. Defining the form factors $g_{\mathbf{m}}$ and the graviton Kaluza-Klein modes $h_{\mu\nu}^{\mathbf{m}}$ as (for simplicity here we compactify on circles)

$$B(\mathbf{y}) = \sum_{\mathbf{m}} e^{-i\mathbf{m}\mathbf{y}} g_{\mathbf{m}} , \quad h_{\mu\nu}(x, \mathbf{y}) = \sum_{\mathbf{m}} e^{i\mathbf{m}\mathbf{y}} h_{\mu\nu}^{(\mathbf{m})}(x) , \quad (220)$$

from the KK expansion, we find

$$\mathcal{L}_{int} = \sum_{\mathbf{m}} \int d^4x \, g_{\mathbf{m}} h_{\mu\nu}^{(\mathbf{m})}(x) T^{\mu\nu}(x) . \quad (221)$$

The “thin brane” approximation $g_{\mathbf{m}} \equiv g = \text{cst}$, or equivalently $B(\mathbf{y}) \sim \delta(\mathbf{y})$ is widely used in the phenomenological literature. This however leads to UV divergences in virtual processes for $\delta \geq 2$ coming from KK states of very large mass. A typical procedure to deal with this difficulty, justified by field-theory considerations [123] or by the analogy with heterotic form factors [81], is to suppress the interactions with heavy KK gravitons introducing a form factor $g_{\mathbf{m}}$ or, equivalently, a the brane thicknes $B(\mathbf{y})$, of the form

$$g_{\mathbf{m}} \sim e^{-\frac{a\mathbf{m}^2}{R^2 M_I^2}} , \quad B(\mathbf{y}) \sim \left(\frac{\pi R^2 M_I^2}{a}\right)^{\frac{\delta}{2}} e^{-\frac{\delta R^2 M_I^2 \mathbf{y}^2}{a}} , \quad (222)$$

where a is a (possibly dependent on the string coupling) constant whose value depends on the model. One of the main purposes of this Section is to compute the D-brane string analog of form factors. It will be shown in particular that (222) reproduces only the *on-shell* string form factor. We will find that its off-shell extension is nonlocal and has a different form (see (232) below), that does not regularize the divergences of virtual gravitational exchange. The resolution of this apparent puzzle, that will be described in the second part of this section, is that in the Type I string this divergence is actually an IR divergence and *not* an UV one. Therefore, string theory does not regulate these divergences, that should instead be cured by the usual procedures the IR divergences are treated in field theory.

- Emission of real gravitons

In the first part of this Section we discuss tree-level string amplitudes with three gauge bosons and one internal (massive) excitation of the graviton. For theories with low string scale and (sub)millimeter dimensions, this type of process is one of the best signals for future accelerators and was studied in field theory in [101]. Schematically, the amplitude for the emission of one massive graviton in field theory is Planck suppressed $1/M_P$. The

inclusive cross section for the emission of gravitons of mass less than the characteristic energy scale of the process E is then proportional to

$$\sigma_{FT} \sim \frac{1}{M_P^2} \sum_{m_i=0}^{RE} 1 \sim \frac{E^\delta}{M^{2+\delta}}, \quad (223)$$

where in the last line we have used the relation $M_P^2 \sim R^\delta M^{2+\delta}$. In (223) M is the effective Planck scale of the higher-dimensional theory, used in most phenomenological papers on the subject, whose relation to the string scale M_I in toroidal compactifications is

$$M/M_I = (1/\pi)^{1/8} \alpha^{-1/4}, \quad (224)$$

where $\alpha = g^2/(4\pi)$ and g is the gauge coupling. Therefore, taking the electromagnetic and the strong coupling as extreme values, we find $1.6 \leq M/M_I \leq 3$. The main point of (223) is that in the inclusive cross section the Planck mass suppression for the emission of each massive graviton is compensated by the large number $(RE)^\delta$ of gravitons kinematically accessible. As a consequence, for energies E close to M , this process could provide an experimental test/signal of models with a low string scale.

A full string formula is needed, however, for energies close to the string scale M_I , where string effects are important and the amplitude (223) violates unitarity. Moreover, as shown in [101], the signal of graviton emission dominates over the Standard Model background for energies $E \geq (0.5 - 0.3)M$, so that the interesting case is $E \geq M_I$, where string effects play an important role in the experimental signal and therefore cannot be ignored²³.

The string amplitude involves the correlation function of three gauge vertex operators, of polarisations ϵ_i and momenta p_i ($i = 1, 2, 3$), and of a massive winding-type graviton of polarisation ϵ_4 and momentum p_4 (see Figure 4). Defining for convenience the Mandelstam

²³Recently, the mixing between Higgs and massive gravitons was proposed as a signal with very small string corrections, provided a term of the form $\xi R H^2$ term exist in the Lagrangian, where R is the scalar curvature tensor and H is the Higgs scalar [102]. The string computation of this operator involves one bulk and two brane fields and can be done along the lines of those performed in this paragraph.

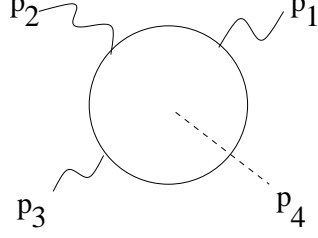


Figure 4: The disk amplitude with three open string particles and one closed string particle.

variables

$$s = -(p_1 + p_2)^2, \quad t = -(p_1 + p_4)^2, \quad u = -(p_1 + p_3)^2, \quad (225)$$

the kinematics of the amplitude is summarized by the equations

$$\begin{aligned} s &= -2p_1p_2 = -2p_3p_4 + w^2, \quad t = -2p_2p_3 = -2p_1p_4 + w^2, \\ u &= -2p_1p_3 = -2p_2p_4 + w^2, \quad s + t + u = w^2. \end{aligned} \quad (226)$$

The details of the computation are given in [127], [128]. As shown there, the final result can be cast in the form

$$A_4 = \frac{1}{\sqrt{\pi}} 2^{-\frac{w^2}{M_I^2}} \frac{\Gamma(-w^2/2M_I^2 + 1/2) \Gamma(-s/2M_I^2 + 1) \Gamma(-t/2M_I^2 + 1) \Gamma(-u/2M_I^2 + 1)}{\Gamma((s - w^2)/2M_I^2 + 1) \Gamma((t - w^2)/2M_I^2 + 1) \Gamma((u - w^2)/2M_I^2 + 1)} A_4^{FT}, \quad (227)$$

where A_4^{FT} is the field-theory amplitude [101]. Eq. (227) obviously reduces to the field theory result in the limit of low energy ($s, t, u \ll M_I^2$) and small graviton mass ($w^2 \ll M_I^2$). This string amplitude has poles for $s, t, u = (2n - 2)M_I^2$, with n a positive integer, corresponding to tree-level open string state exchanges in the s , t and u channels. Moreover, there are poles for graviton winding masses $w^2 = (2n - 1)M_I^2$, to be interpreted as tree-level mixings between the graviton and the gauge singlets present at odd levels in the open string spectrum, which then couple to the gauge fields. The amplitude A_4 has also zeroes for very heavy gravitons $w^2 = s + 2nM_I^2$, or similar conditions obtained by the replacements $s \rightarrow t, u$. These give interesting selection rules and display a typical behavior, not shared by any other field theory process with missing energy.

An important question raised by (227) concerns the string deviations from the field theory result A_4^{FT} . In the s-channel, the energy corresponding to the first string resonance is $s = 2M_I^2$, and similarly for t and u . This supports the natural expectation that field theory breaks down for energies above M_I . For energies well below this value ($s, t, u, w^2 \ll M_I^2$), it is easy to obtain the corrections to the field-theory computation from a power-series expansion of (227). The first corrections are of the form

$$A_4 = (1 + \frac{\zeta(2)}{4} \frac{w^4}{M_I^4} + \frac{\zeta(3)}{4} \frac{stu + w^6}{M_I^6} + \dots) A_4^{FT} \quad (228)$$

and, after T-duality become

$$A_4 = (1 + \frac{\pi^2}{24} \frac{m^4}{(R_\perp M_I)^4} + \dots) A_4^{FT} . \quad (229)$$

Notice that the first correction to the amplitude with a massless graviton (of fixed energy) is of order²⁴ E^6/M_I^6 , and therefore the deviation from the field theoretical result is first expected to arise from massive gravitons.

A more useful way to define deviations from field theory is the integrated cross-section σ , obtained summing over all graviton masses, up to the available energy E

$$\sigma = \sum_{m_1 \dots m_6=0}^{R_\perp E} |A_4|^2 \quad , \quad \sigma^{FT} = \sum_{m_1 \dots m_6=0}^{R_\perp E} |A_4^{FT}|^2 \quad , \quad (230)$$

where σ^{FT} is the corresponding field theory value. Surprisingly enough, terms of order E^2 are absent in (230) and therefore at low energies the string corrections are smaller than expected, of order

$$\frac{\sigma - \sigma^{FT}}{\sigma^{FT}} \sim \frac{E^4}{M_I^4} . \quad (231)$$

However, as mentioned above, strong deviations emerge for $E^2 \sim 2M_I^2$, where the first string resonance appears and the field theory approach breaks down.

Another interesting quantity is the form-factor for two gauge bosons and one winding (KK mode \mathbf{m} after T-dualities) graviton, or, in a more phenomenological language, of

²⁴This is probably related to the underlying $\mathcal{N} = 4$ supersymmetry of this toroidal compactification.

the bulk/brane/brane couplings, which were already used in previous sections to discuss neutrino (and axion) masses. A direct on-shell computation can easily be done [125], and the result turns out to be the same in the bosonic string and in the superstring [127]. A partly off-shell expression for the form factor can however be obtained factorizing the three gauge bosons – one massive graviton amplitude. Indeed, using (227) and the two gauge bosons – one massive graviton amplitude [125, 127], it is possible to find the form factor in the case where one of the gauge bosons and the graviton are off-shell²⁵ [127]

$$g(p_1, p_2, p) = \frac{1}{M_P \sqrt{\pi}} 2^{\frac{p^2}{M_I^2}} \frac{\Gamma(p^2/2M_I^2 + 1/2)}{\Gamma(p_1 p_2/M_I^2 + 1)} . \quad (232)$$

Notice that, at energies much smaller than the string scale ($p^2, p_1 p_2 \ll M_I^2$), this form factor is close to $1/M_P$ for all winding states, a result that was used in the field theory approach to the brane/brane/bulk couplings in Section 11.

From (232) we can deduce a form factor characterizing the emission of a heavy graviton ($-p^2 \gg M_I^2$)

$$g(p^2) \sim 2 \sqrt{\frac{2M_I^2}{\pi p^2}} \left(\tan \frac{\pi p^2}{M_I^2} \right) e^{\frac{p^2}{M_I^2} \ln 2} , \quad (233)$$

where for an on-shell graviton p^2 is equal to the KK graviton mass $-p^2 = m^2/R_\perp^2$. It is transparent from this result that we qualitatively recover, aside from an oscillatory factor accounting for the string resonances, the field-theory exponential form factor (222), but only for *on-shell* particles. Indeed, the off-shell result (232) contains, as expected, the exponential damping of string amplitudes at high-energy [128] in the fixed angle limit, irrespective of the graviton KK mass.

- Virtual exchange of string and gravitational-type states

One of the main possible experimental signatures for String Theory is the tree-level exchange of string oscillators (Regge particles), encoded in the four-particle Veneziano

²⁵The expression (232) corrects a factor 2 misprint in the eqs. (1.9) and (4.16) of [127].

amplitude

$$A(s, t) = \frac{\Gamma(1 - s/M_I^2)\Gamma(1 - t/M_I^2)}{\Gamma(1 - s/M_I^2 - t/M_I^2)} , \quad (234)$$

and in similar expressions $A(t, u)$, $A(u, s)$, that have poles corresponding to massive open string states. They manifest themselves as deviations from field theory amplitudes for energies close to the string scale M_I . Moreover, they produce an exponential damping $\exp(-s/M_I^2)$ of the amplitudes at high energy $s \gg M_I^2$, for scatterings at fixed angle.

The corresponding one-loop diagrams have a dual interpretation as tree-level virtual exchanges of gravitational-type particles. In a field-theory approach, these contributions have the problem that for a number of compact dimensions $d \geq 2$ the corresponding KK field theory summations diverge in the ultraviolet (UV), and therefore the field-theory computation is unreliable. Indeed, let us consider a four-fermion interaction of particles confined to a D3 brane mediated by KK gravitational excitations orthogonal to it. The amplitude for the process reads

$$A = \frac{1}{M_P^2} \sum_{m_i} \frac{1}{-s + \frac{m_1^2 + \dots + m_\delta^2}{R_\perp^2}} , \quad (235)$$

where for simplicity we considered equal radii denoted by R_\perp and $s = -(p_1 + p_2)^2$ is the squared center of mass energy²⁶. The summation clearly diverges for $\delta \geq 2$. In this cas, the traditional attitude is to cut the sums for masses heavier than a cutoff $\Lambda \gg R_\perp^{-1}$, of the order of the fundamental scale M_I in string theory [101]. This can be implemented in a proper-time representation of the amplitude

$$A = \frac{1}{M_P^2} \sum_{m_i} \int_{1/\Lambda^2}^\infty dl e^{-l(-s + \frac{m_1^2 + \dots + m_\delta^2}{R_\perp^2})} = \frac{1}{M_P^2} \int_{1/\Lambda^2}^\infty dl e^{sl} \theta_3^\delta(0, \frac{il}{\pi R_\perp^2}) , \quad (236)$$

where $\theta_3(0, \tau) = \sum_k \exp(i\pi k^2 \tau)$ is one of the Jacobi functions. We shall be interested in the following in the region of parameter space $-R_\perp^2 s \gg 1$, $R_\perp \Lambda \gg 1$ and $-s \ll \Lambda^2$, in which the available energy is smaller than (but not far from) the UV cutoff Λ , but is

²⁶With our conventions s is negative in Euclidean space.

much bigger than the (inverse) compact radius R_\perp^{-1} , of submillimeter size. In this case, the amplitude can be evaluated and is

$$A = \frac{\pi^\delta R_\perp^\delta}{M_P^2} \int_{1/\Lambda^2}^\infty \frac{dl}{l^{\frac{\delta}{2}}} e^{sl} \theta_3^\delta(0, \frac{i\pi R_\perp^2}{l}) \simeq \frac{2\pi^{\frac{\delta}{2}} R_\perp^\delta \Lambda^{\delta-2}}{\delta-2 M_P^2} = \frac{4\pi^{\frac{\delta}{2}} \alpha_{YM}^2 \Lambda^{\delta-2}}{\delta-2 M_I^{\delta+2}}, \quad (237)$$

where in the last step we used the relation $M_P^2 = (2/\alpha_{YM}^2) R_\perp^\delta M_I^{2+\delta}$, valid for Type I strings, where $\alpha_G = g_{YM}^2/(4\pi)$ and g_{YM} is the Yang-Mills coupling on our brane. The cutoff Λ is equivalent to computing the field-theory diagram using a form factor of the type (222) with $\Lambda = M_I/\sqrt{a}$. As shown at the beginning of this section, however, (222) is an on-shell form factor in string theory, whereas virtual particle exchanges ask for an off-shell form factor. The off-shell form factor (232), on the other hand, depends only on the momentum of the massive gravitons and *not* on its mass. This therefore raises doubts on the way string theory regulates the divergent sum (235). In addition, the high sensitivity of the result (237) to the cutoff Λ asks for a more precise computation in a full string context. As explained below, string theory does not cut the divergent sum (223). The solution to the puzzle is that the divergent sum is not an UV divergence in string theory, but an IR divergence, which has the same status in string and in field theory, and asks for resummation of graphs with soft particle emissions²⁷.

The relevant string diagram is actually one loop and is a priori subdominant with respect to tree-level Veneziano amplitudes. However, deviations from Newton's law come precisely from this one-loop diagram, and therefore a precise evaluation is necessary.

The computation reviewed below was done for the $SO(32)$ Type I 10D superstring compactified to 4D on a six-dimensional torus. However, we will argue later that the result

²⁷Another interesting, related example of this type of divergence, arises in the one-loop effective action of toroidal compactifications [75]. Indeed, there are F^4 terms on the Type I and on the heterotic $SO(32)$ side that match using the Type I-heterotic duality relations (5). However, the coefficient of the corresponding terms is proportional to $\sum_{\mathbf{m} \neq \mathbf{0}} (1/\mathbf{m}^2)$, where $\mathbf{m} = (m_1 \cdots m_6)$. This sum is divergent due to the contribution of very heavy KK states, as the amplitude (235). The Type I-heterotic duality check performed in [75] then holds with an appropriate identification of the IR cutoffs on the two sides.

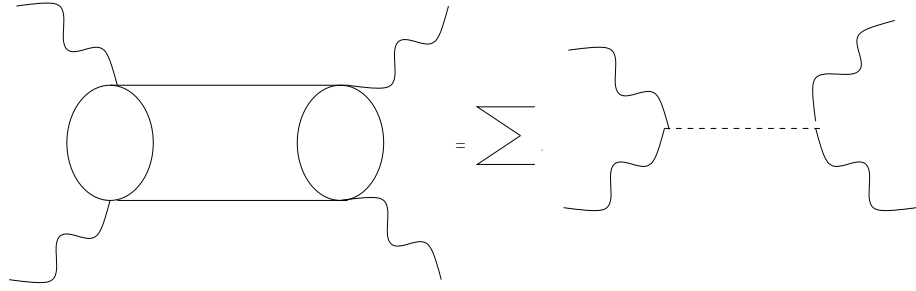


Figure 5: The closed channel representation of the amplitude.

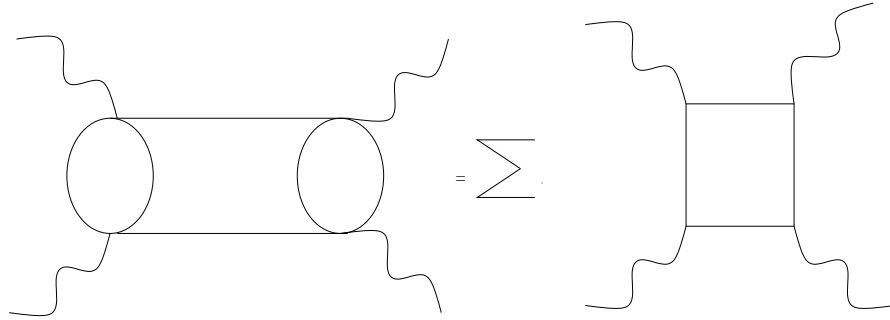


Figure 6: The open channel representation of the amplitude.

holds for a large class of orbifolds, including $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetric vacua. The Type I string diagram that in the low-energy limit contains the gravitational exchange mentioned above is the nonplanar cylinder diagram, in which for simplicity we prefer to insert gauge bosons rather than fermions in the external lines. This diagram has two dual interpretations [124] a) tree-level exchange of closed-string states, if time is chosen to run horizontally (see Fig. 5) b) one-loop diagram of open strings, if time is chosen to run vertically (see Fig. 6). In the two dual representations, the nonplanar amplitude reads symbolically

$$\begin{aligned}
 A &= \sum_n \int_0^\infty dl \sum_{n_i} A_2(l, n_1 \cdots n_d, n) \\
 &= \sum_{k_1 \cdots k_d} \int_0^\infty d\tau_2 \tau_2^{d/2-2} \sum_{m_i} A_1(\tau_2, m_1 \cdots m_d, k_1 \cdots k_d) ,
 \end{aligned} \tag{238}$$

where l denotes the cylinder parameter in the tree-level channel and $\tau_2 = 1/l$ is the one-loop open string parameter²⁸. In the first representation, the amplitude is interpreted as tree-level exchange of closed-string particles of mass $(n_1^2 + \cdots n_d^2)R^2 M_I^4 + n M_I^2$, where $n_1 \cdots n_d$

²⁸ τ_2 was called t in previous chapters. In this paragraph, however, we reserve the symbol t for one of the

are winding quantum numbers and n is the string oscillator number. In particular the $n = 0$ term reproduces the field-theory result (235), and therefore the full expression (238) is its string regularization. In the second representation, the amplitude is interpreted as a sum of box diagrams with particles of masses $(m_1^2 + \cdots m_d^2)/R^2 + k_i M_I^2$ ($i = 1 \cdots 4$) in its four propagators.

The UV limit ($l \rightarrow 0$) of the gravitational tree-level diagram is related to the IR limit ($\tau_2 \rightarrow \infty$) of the box diagram. In particular, in four dimensions, when an IR regulator μ is introduced in the box diagram, the divergence in the Kaluza-Klein summation in the gravitational-exchange diagram disappears. The final result for the nonplanar cylinder amplitude in the low energy limit $E/M_I \ll 1$ (E is a typical energy scale) in four-dimensions is [127]

$$\begin{aligned} A = & -\frac{1}{\pi M_P^2 s} + \frac{2g_{YM}^4}{\pi^2} \left[\frac{1}{st} \ln \frac{-s}{4\mu^2} \ln \frac{-t}{4\mu^2} + \text{perms.} \right] \\ & - \frac{g_{YM}^4}{3M_I^4} \left[\ln \frac{s}{t} \ln \frac{st}{\mu^4} + \ln \frac{s}{u} \ln \frac{su}{\mu^4} \right] + \cdots , \end{aligned} \quad (239)$$

where “perms.” denotes two additional contributions coming from permutations of s, t, u and “ \cdots ” denote terms of higher order in the low energy expansion. Notice the absence in (239) of the contact term (237), that in the string result is replaced by the leading string correction, given by the second line in (239). The string correction in (239) is indeed of the same order of magnitude as (237) for $\Lambda \sim M_I$, but has an explicit energy dependence coming from the logarithmic terms.

In order to find the appropriate interpretation of (239) in terms of field-theory diagrams, it is convenient to separate the integration region in (238) into two parts, introducing an arbitrary parameter l_0 and writing

$$A = \sum_n \int_{l_0}^{\infty} dl \sum_{n_i} A_2 + \sum_{k_1 \cdots k_4} \int_{1/l_0}^{\infty} d\tau_2 \tau_2^{d/2-2} \sum_{m_i} A_1 . \quad (240)$$

Mandelstam variables.

This has the effect of fixing an UV cutoff $\Lambda = M_I/\sqrt{l_0}$ in the tree-level exchange diagram, similar to the one introduced in (236) and (237), as well as a related UV cutoff $\Lambda' = M_I\sqrt{l_0} = M_I^2/\Lambda$ in the one-loop box diagram described here by A_1 . Computing the low-energy limit of A_1 and A_2 , in 4d we find [127]

$$\begin{aligned} A_1 &= \frac{2g_{YM}^4}{\pi^2} \left[\frac{1}{st} \ln \frac{-s}{4\mu^2} \ln \frac{-t}{4\mu^2} + \text{perms.} \right] - \frac{g_{YM}^4}{3M_I^4} \left[\ln \frac{s}{t} \ln \frac{st}{\mu^4} + \ln \frac{s}{u} \ln \frac{su}{\mu^4} + \frac{6}{l_0^2} \right] + \dots \\ A_2 &= -\frac{1}{\pi M_{Ps}^2} + \frac{2g_{YM}^4}{M_I^4} \left[\frac{1}{l_0^2} + \dots + O\left(\frac{s^2}{M_I^4}\right) + \dots \right]. \end{aligned} \quad (241)$$

The g_{YM}^4 term in A_1 describes a box diagram with four light particles (of mass μ) circulating in the loop, while the g_{YM}^4/M_I^4 term is the first string correction coming from box diagrams with one massive particle (of mass M_I) and three light particles of mass μ in the loop. It also contains the l_0 dependent part of the box diagram with four light particles in the loop. The $1/M_I^4 l_0^2$ term in A_2 can be written as Λ^4/M_I^8 and therefore reproduces the field theory result (237) in the case of six compact dimensions. However, as expected, a similar term with an opposite sign appears in A_1 , while the l_0 dependent terms cancel. In A_2 the $O(s^2/M_I^4)$ term is l_0 independent, and is actually the first correction to the tree-level graviton exchange. We emphasize, however, that the only physically meaningful amplitude is the full expression (239). The leading string correction is therefore the second line of (239), coming from box diagrams A_1 involving one massive particle in the loop.

Strictly speaking, this result is valid for toroidal compactifications of the $SO(32)$ 10D Type I string. For a general $\mathcal{N} = 1$ supersymmetric 4D Type I vacuum the amplitude A has contributions from sectors with various numbers of supersymmetries

$$A = A^{\mathcal{N}=4} + A^{\mathcal{N}=2} + A^{\mathcal{N}=1}, \quad (242)$$

where the $\mathcal{N} = 4$ sector contains the six-dimensional compact KK summations, the $\mathcal{N} = 2$ sectors contain two-dimensional compact KK summations and the $\mathcal{N} = 1$ sectors contain no KK summations. From the tree-level (A_2) viewpoint, the $\mathcal{N} = 2$ sectors give logarithmic divergences that in the one-loop box (A_1) picture correspond to additional infrared diver-

gences associated to wave-functions or vertex corrections, absent (by nonrenormalization theorems) in the $\mathcal{N} = 4$ theory. Similarly, the $\mathcal{N} = 1$ sectors give no KK divergences. As the important (power-like) divergences come from the gravitational $\mathcal{N} = 4$ sector, the toroidally compactified Type I superstring contains the relevant information for our purposes. Moreover, even if we confine our attention to the Type I superstring, the formalism can be easily adapted to Type II strings and to their D-branes. This can be done exchanging some of the Neumann boundary conditions in the compactified Type I string with the Dirichlet ones appropriate for the D-branes [125, 126]. As can be easily seen, the basic results and conclusions of this Section are unchanged.

An interesting observation was made recently concerning models where some of the Standard Model fermions live on a brane, while others live on a distant brane. In this case, all amplitudes for processes involving fermions on the two branes have an exponential damping factor depending on the distance between the branes [129], that would produce spectacular effects in accelerator experiments.

13. Conclusions

The last years had a dramatic effect on our understanding of string physics and of its possible implications for low energy physics. In particular, there is a real hope to experimentally test scenarios with a low string scale, large compactification (TeV) radii and possibly (sub)millimeter gravitational dimensions. Some of the relevant issues (gauge coupling unification, supersymmetry breaking, gauge hierarchy) were already analyzed at the string level using quasirealistic string models, while other issues (flavor physics, for example) were mainly studied at the field theory level, so that more detailed string studies would be very useful. This review does not cover cosmological issues (see, for example, [130] and references therein) and the recent scenarios related to warped compactifications [131] (for the role of warped compactifications in strings, see for example [58, 133]). In

particular, the last scenarios provide the first phenomenological models of Anti-de-Sitter compactifications, which led to the famous AdS/CFT conjecture [132] with interesting, nonperturbative results, for the gauge theory dynamics.

It is however important to keep in mind that, despite the beautiful new ideas dealing with large (or infinite) extra dimensions which appeared recently, the good old picture of the “desert” between the weak scale and a large (of the order of 10^{16} GeV) unification scale is still a viable possibility. Since String Theory at the present time offers no compelling reason in favor of any of the new scenarios, only new experimental results can provide a hint for the real value of the string scale or, more generally, for the correct picture of physics beyond the Standard Model.

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A Jacobi functions, lattice sums and their properties

For the reader’s convenience, in this Appendix we collect the definitions, transformation properties and some identities for the modular functions used in the text (for more formulae and properties of modular functions, see for example [134]). The Dedekind function is

defined by the usual product formula (with $q = e^{2\pi i\tau}$)

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) , \quad (243)$$

whereas the Jacobi ϑ -functions with general characteristic and arguments are

$$\vartheta\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}\right](z, \tau) = \sum_{n \in \mathbb{Z}} e^{i\pi\tau(n-\alpha)^2} e^{2\pi i(z-\beta)(n-\alpha)} . \quad (244)$$

The corresponding product representation of the Jacobi functions is

$$\vartheta\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}\right](z, \tau) = e^{2\pi i\alpha(\beta-z)} q^{\frac{\alpha^2}{2}} \prod_{n=1}^{\infty} (1 - q^n) [1 + q^{n-\alpha-\frac{1}{2}} e^{2\pi i(z-\beta)}] [1 + q^{n+\alpha-\frac{1}{2}} e^{-2\pi i(z-\beta)}] . \quad (245)$$

For completeness, we give also the product formulae for the four special ϑ -functions with half-integer characteristics

$$\begin{aligned} \vartheta_1(z, \tau) &\equiv \vartheta\left[\begin{smallmatrix} \frac{1}{2} \\ \frac{1}{2} \end{smallmatrix}\right](z, \tau) = 2q^{1/8} \sin \pi z \prod_{n=1}^{\infty} (1 - q^n) (1 - q^n e^{2\pi i z}) (1 - q^n e^{-2\pi i z}) , \\ \vartheta_2(z, \tau) &\equiv \vartheta\left[\begin{smallmatrix} \frac{1}{2} \\ 0 \end{smallmatrix}\right](z, \tau) = 2q^{1/8} \cos \pi z \prod_{n=1}^{\infty} (1 - q^n) (1 + q^n e^{2\pi i z}) (1 + q^n e^{-2\pi i z}) , \\ \vartheta_3(z, \tau) &\equiv \vartheta\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right](z, \tau) = \prod_{n=1}^{\infty} (1 - q^n) (1 + q^{n-1/2} e^{2\pi i z}) (1 + q^{n-1/2} e^{-2\pi i z}) , \\ \vartheta_4(z, \tau) &\equiv \vartheta\left[\begin{smallmatrix} 0 \\ \frac{1}{2} \end{smallmatrix}\right](z, \tau) = \prod_{n=1}^{\infty} (1 - q^n) (1 - q^{n-1/2} e^{2\pi i z}) (1 - q^{n-1/2} e^{-2\pi i z}) . \end{aligned} \quad (246)$$

The modular properties of these functions are described by

$$\eta(\tau + 1) = e^{i\pi/12} \eta(\tau) , \quad \vartheta\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}\right](z, \tau + 1) = e^{-i\pi\alpha(\alpha-1)} \vartheta\left[\begin{smallmatrix} \alpha \\ \alpha + \beta - \frac{1}{2} \end{smallmatrix}\right](z, \tau) \quad (247)$$

$$\eta(-1/\tau) = \sqrt{-i\tau} \eta(\tau) , \quad \vartheta\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}\right]\left(\frac{z}{\tau}, \frac{-1}{\tau}\right) = \sqrt{-i\tau} e^{2i\pi\alpha\beta + i\pi z^2/\tau} \vartheta\left[\begin{smallmatrix} \beta \\ -\alpha \end{smallmatrix}\right](z, \tau) . \quad (248)$$

The relevant Kaluza-Klein and winding lattice summations appearing in the text are

$$\sum_m P_{m+a}(\tau) \equiv \sum_m q^{\frac{\pi\alpha'(m+a)^2}{R^2}} , \quad \sum_n W_{n+b}(\tau) \equiv \sum_n q^{\frac{\pi}{4\alpha'}(n+b)^2 R^2} , \quad (249)$$

with $q = e^{2\pi i\tau}$, where in the summations relevant to the D9 branes, for example, $\tau = it/2$ in P_{m+a} and $\tau = il$ in W_{n+b} . A Poisson formula used frequently in order to pass from the

one loop open-string channel to the tree-level closed string channel is

$$\sum_m e^{-\pi\alpha' t \frac{(m+a)^2}{R^2}} = \left(\frac{R^2 l}{2\alpha'}\right)^{\frac{1}{2}} \sum_n e^{2\pi i a n} e^{-\frac{\pi l}{2\alpha'} n^2 R^2}, \quad (250)$$

where $t/2 = 1/l$.

B Glossary

- **Chan-Paton factors:** quantum numbers which sit at the end of open strings, which give rise to the gauge group and the charged matter quantum numbers.

- **critical dimension:** spacetime dimension (10 for superstrings) in which the 2d Weyl anomaly cancels and the world-sheet theory has a background which is Poincaré invariant.

- **Dp-brane:** dynamical surface which spans $p + 1$ spacetime dimensions, containing gauge group and charged matter, on which open strings (with Dirichlet boundary conditions) can end. D-branes are embedded in an underlying space of dimension ten for superstrings.

- **Green-Schwarz mechanism:** gauge and gravitational anomaly cancellation in 10d due to the nonlinear gauge transformation of the antisymmetric tensor field.

- **GSO projection:** projection of physical states which enforces modular invariance.

- **modular invariance:** invariance of the one-loop partition function of closed strings under global reparametrizations of the torus.

- **no-scale model:** supergravity models with (tree-level) zero vacuum energy and broken supersymmetry, having flat directions in the scalar potential, along which the size of supersymmetry breaking is classically undetermined.

- **orbifolds:** compact spaces of the type M/G , where G is a discrete group, on which string propagation can be exactly solved. Supersymmetry is generically partly or completely broken, such that the method can generate chiral models in four-dimensions.

- **orientifolds:** String models constructed by gauging world-sheet symmetries H (for example the world-sheet parity Ω), such that physical states are H invariant. The topological expansion in this case involves non-orientable surfaces (e.g. Klein bottle or Möbius strip).

- **orientifold (O) plane:** fixed (non-dynamical) surface under the orientifold projection, carrying Ramond-Ramond charges.

- **partition function:** the vacuum-energy at a given order in the topological expansion.

- **Ramond-Ramond fields:** Antisymmetric tensor fields of different rank present in the closed string spectrum, which couple to orientifold planes and D-branes.

- **Scherk-Schwarz mechanism:** breaking of supersymmetry due to boundary conditions in the compact space, different for bosons and fermions.

- **vertex operators:** operators constructed out of world-sheet degrees of freedom, used in constructing string correlations functions for external on-shell particles.

- **Wilson line:** gauge field in compact space with vanishing field strength. The wave function of charged states acquires a phase after a closed loop in the compact direction. The corresponding states become massive and break the gauge group to a subgroup.

- **winding state:** massive state coming from closed strings wrapping a compact coordinate. A string wrapping n times a circle of radius R gives a mass $n RM_s^2$.

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